Intro Physics I EXAM 1
2012 February 1, Wednesday NAME:

Instructions: There are 20 multiple-choice problems each worth 1 mark for a total of 20 marks altogether. Choose the **BEST** answer, completion, etc., and **DARKEN** fully the appropriate circle on the table provided below. Read all responses carefully. **NOTE** long detailed responses won't depend on hidden keywords: keywords in such responses are bold-faced capitalized. Harder/longer multiple-choice problems (which are sometimes based on full-answer problems) are marked by asterisks: $*$ for easy, $**$ for moderate, $***$ for hard: all in the judgment of the instructor.

There are **THREE** full-answer problems worth 10 marks for a total of 30 marks altogether. Answer them all. It is important that you **SHOW** (**SHOW, SHOW, SHOW**) how you got the answer for the full-answer problem. Don't give up on problems where you can't do the first part: sometimes later parts can be done independently. Some full-answer problems may be multiple-pagers: make sure you have answered everything. And **BOX-IN** your final answers.

This is a **CLOSED-BOOK** exam. **NO** cheat sheets allowed. An **EQUATION SHEET** is provided. Calculators are permitted—but **ONLY** for calculations. There are **SCRATCH PAGES** for auxiliary calculations. Remember your name (and write it down on the exam too).

The exam is out of 50 marks altogether and is a 50-minute exam.

Answer Table for the Multiple-Choice Questions

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	11.	O	O	O	O	O
2.	O	O	O	O	O	12.	O	O	O	O	O
3.	O	O	O	O	O	13.	O	O	O	O	O
4.	O	O	O	O	O	14.	O	O	O	O	O
5.	O	O	O	O	O	15.	O	O	O	O	O
6.	O	O	O	O	O	16.	O	O	O	O	O
7.	O	O	O	O	O	17.	O	O	O	O	O
8.	O	O	O	O	O	18.	O	O	O	O	O
9.	O	O	O	O	O	19.	O	O	O	O	O
10.	O	O	O	O	O	20.	O	O	O	O	O

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001 qmult 00300 1 4 2 easy deducto-memory: scientific notation defined

Extra keywords: physci

1. "Let's play *Jeopardy*! For $100, the answer is: It is a notation in which one expresses a number by a coefficient decimal number multiplied explicitly by 10 to the appropriate power. If the coefficient is in the range 1 to 10, but not including 10, the notation is called normalized."

   What is ____________, Alex?

   a) British notation b) scientific notation c) metric notation d) tensy notation e) Irish notation

   SUGGESTED ANSWER: (b)

   Wrong answers:

   e) Tis yourself that does not know that scientific notation was invented by St. Patrick who, alas, was actually British.

   Redaction: Jeffery, 2001jan01

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001 qmult 00424 1 1 5 easy memory: US Customary Unit System 2

2. The 2nd largest country in North America in terms of land area for most everyday purposes uses the system of:

   a) brutish units—brut, not brit. b) Finnish units. c) Pictish units. d) kludgy units. e) US Customary units.

   SUGGESTED ANSWER: (e)

   Wrong answers:

   d) Arguably so.

   Redaction: Jeffery, 2008jan01

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001 qmult 00560 1 4 3 easy deducto-memory: conversion mile to km

3. 1 mile is nearly exactly:

   a) 1 m. b) 1 km. c) 1.609 km. d) 2.54 cm. e) 1 ft.

   SUGGESTED ANSWER: (c)

   You can deduce the answer. How likely is it that 1 mi would nearly exactly 1 km?

   Actually, the modern mile is defined to be exactly 1.609344 km (http://en.wikipedia.org/wiki/Mile).

   Wrong answers:

   a) A meter is about a yard=36 inches.
   b) Doesn't seem likely given generally offset between SI and British units.
   d) Not too likely
   e) Oh c'mon.

   Redaction: Jeffery, 2001jan01

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001 qmult 00710 1 3 3 easy math: sig.fig. multiplication

4. Multiply experimental values 3.27 and 9.9, and give the answer to correct significant figures.

   a) 30.   b) 34.   c) 32.   d) 32.3.   e) 32.4.

   SUGGESTED ANSWER: (c)

   Behold:

   $$3.27 \times 9.9 = 32.373$$

   exactly, but rounded off to significant figures one has 32.

   Fortran-95 Code

```
print*
x=3.27d0
y=9.9d0
print*,'x*y=',x*y  !  32.3730000000000
```

   Wrong answers:

   b) This isn't even correctly rounded off.

   d) This isn't even correctly rounded off.

   Redaction: Jeffery, 2001jan01

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001 qmult 10510 2 3 5 moderate math: acre-foot conversion fullmult

5.** American hydraulic engineers often use acre-feet to measure volume of water. An acre-foot is the amount of water that will cover an acre of flat land to 1 foot. Say 4.00 in of rain fell on a plain of 30.0 km^2. How many acre-feet of water fell? Note 1 square mile equals 640 acres and 1 km = 0.6214 mi. **HINTS:** First, find the volume in the hybrid units of inch-km^2 and then use a separate factor of unity for each unit conversion: divide and conquer.

   a) 1066 acre-feet.   b) 1492 acre-feet.   c) 1776 acre-feet.
   d) 1850 acre-feet.   e) 2470 acre-feet.

   SUGGESTED ANSWER: (e)

   Behold:

   $$4.00\,\text{in} \times 30\,\text{km}^2 = 4.00\,\text{in} \times 30\,\text{km}^2 \times \left(\frac{1\,\text{ft}}{12\,\text{in}}\right) \times \left(\frac{0.6214\,\text{mi}}{1\,\text{km}}\right)^2 \times \left(\frac{640\,\text{acres}}{1\,\text{mi}^2}\right)$$
   $$= 2470\ \text{acre-feet} .$$

   Fortran-95 Code

```
          print*
volume=4.d0*30.d0*(1.d0/12.d0)*(.6214d0/1.d0)**2*(640.d0/1.d0)
          print*,'volume in acre-feet'
          print*,volume
    !    2471.2829439999995
```

Wrong answers:

a) In 1066, William the Conqueror, Duke of Normandy conquered England.
b) In 1492, Columbus sailed the ocean blue.
c) Well,

Listen my children and you shall hear
Of the midnight ride of Paul Revere,
On the eighteenth of April, in Seventy-five;
Hardly a man is now alive
Who remembers that famous day and year.

Old Paul would have understood all about acre-feet.

Redaction: Jeffery, 2008jan01

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002 qmult 00152 1 1 3 easy memory: constant acceleration gives quadratic displacement

6. If acceleration is constant, then velocity is linear with time and displacement is ____________ with time.

a) constant b) linear c) quadratic d) cubic e) quartic

SUGGESTED ANSWER: (c)

Wrong answers:

a) Not likely.

Redaction: Jeffery, 2008jan01

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002 qmult 00322 1 3 2 easy math: average speed on round trip: Knoxville 2
Extra keywords: a round trip to Knoxville

7. You have just traveled a total distance of 400 km on a trip to Knoxville and back. It took 8 hours. Your average **SPEED** was:

a) 0 km/h. b) 50 km/h. c) 100 km/h. d) 200 km/h. e) 400 km/h.

SUGGESTED ANSWER: (b) An easy math question. This is the lead in to an average velocity question.

Actually, there is a subtlety. The average speed is distance traveled divided by time. This is not the magnitude of the average velocity. But in the limit that the time interval goes to zero it becomes that. Thus, instantaneous speed is the magnitude of instantaneous velocity. These definitions are the most conventional ones—they are what people ordinarily mean—I'm taking no arguments.

Wrong answers:

a) This is your average velocity.

Redaction: Jeffery, 2008jan01

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002 qmult 00520 1 1 2 easy memory: zero velocity at top of trajectory

8.* An object is thrown straight up. At the top of its trajectory, it's vertical velocity is:

a) always completely indeterminate. b) zero. c) infinite. d) g.
e) unknown with the given information.

SUGGESTED ANSWER: (b)

Wrong answers:
c) A nonsense answer.

Redaction: Jeffery, 2008jan01

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002 qmult 00620 3 1 2 easy math: travel time, human terminal velocity 1
Extra keywords: physci

9. The terminal velocity of a human in air is about 120 mi/h. At this speed how long does it take to fall 2 miles.

a) 2 minutes. b) 1 minute. c) 1 hour. d) 2 hours. e) 1 second.

SUGGESTED ANSWER: (b)

The students have to be clear on how you get a time from a distance and speed: distance/speed.

In this case

$$\frac{2\,\mathrm{mi}}{120\,\mathrm{mi/h}} = \frac{1}{60}\,\mathrm{h} \times \left(\frac{60\,\mathrm{minutes}}{1\,\mathrm{h}}\right) = 1\,\mathrm{minute}\ .$$

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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002 qmult 10410 2 3 1 moderate math: easy car motion fullmult

10.* A car starts from **REST** with constant acceleration $1\,\mathrm{m/s^2}$.

i) What is its velocity after 10 s from its **START POSITION**?

ii) What is its displacement traveled after 10 s from its **START POSITION**?

iii) What is its velocity when it has traveled 200 m from its **START POSITION**?

a) (i) 10 m/s (ii) 50 m (iii) 20 m/s. b) (i) 10 m/s (ii) 50 m (iii) 14.1 m/s.
c) (i) 10 m/s (ii) 10 m (iii) 10 m/s. d) (i) 10 m/s (ii) 10 m (iii) 14.1 m/s.
e) (i) 10 m/s (ii) 10 m (iii) 20 m/s.

SUGGESTED ANSWER: (a)

i) Behold:

$$v = at = 10\,\mathrm{m/s}\ .$$

ii) Behold:

$$x = \frac{1}{2}at^2 = 50\,\mathrm{m}\ .$$

iii) Behold:

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{400} = 20\,\mathrm{m/s}\ .$$

Wrong answers:

b) This is the answer to the full-answer problem, but I've changed the input values a bit.

Redaction: Jeffery, 2008jan01

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003 qmult 00230 1 1 4 easy memory: definition of acceleration

Extra keywords: physci

11. Acceleration is:

  a) speed.
  b) velocity.
  c) the rate of change of velocity with time. It is a **SCALAR**.
  d) the rate of change of velocity with time. It is a **VECTOR**.
  e) the rate of change of displacement with time. It is a **VECTOR**.

SUGGESTED ANSWER: (d)

Wrong answers:

  c) There is a scalar meaning of acceleration, but that isn't rate of change of velocity and it isn't the first meaning.

Redaction: Jeffery, 2001jan01

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003 qmult 00330 1 1 1 easy memory: basic trig function names

12. The three basic trig functions have abbreviated names:

  a) sin, cos, tan. b) sly, crow, tawn. c) slip, crape, toon.
  d) slop, crip, troop. e) snood, croon, troon.

SUGGESTED ANSWER: (a)

Wrong answers:

  e) It's hard to believe in snood, but it is in the dictionary.

Redaction: Jeffery, 2008jan01

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003 qmult 00610 1 4 2 easy deducto-memory: dot product defined 2

13. "Let's play *Jeopardy*! For $100, the answer is: It is the product of the magnitudes of two vectors multiplied by the cosine of the angle between them."

  What is ____________, Alex?

  a) trot product b) dot product c) vector product d) cross product
  e) angry product

SUGGESTED ANSWER: (b)

Wrong answers:

  c) This is another kind of vector product.
  d) This is another for the vector product.
  e) This very cross product.

Redaction: Jeffery, 2008jan01

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003 qmult 00822 1 3 2 easy math: displacements in Vegas 2

Extra keywords: physci

14. You are in Las Vegas where the streets are almost all laid out in a rectangular grid. Assume the grid is exactly aligned with the cardinal directions. You drive 2 miles north on the Strip, turn right on Flamingo at the Harley-Davidson Cafe, drive east 3 miles to Maryland, turn south on Maryland and drive 1 mile to Tropicana, and, finally, turn right on Tropicana and drive 3 miles. Where are you? How many miles have traveled? What is your total **DISPLACEMENT**? **HINT:** Draw a diagram.

a) On the Strip (at the MGM Grand for what it's worth), 9 miles, and **1 MILE**.
b) On the Strip (at the MGM Grand for what it's worth), 9 miles, and **1 MILE NORTH**.
c) On the Strip (at the MGM Grand for what it's worth), 1 mile, and **9 MILES NORTH**.
d) On the Strip (at the MGM Grand for what it's worth), 1 mile, and **9 MILES**.
e) On the Strip (at the Hard Rock Cafe), 9 miles, and **9 MILES**.

SUGGESTED ANSWER: (b)

This is the trick question since its so tempting to stop at answer (a).

Ah, I remember the corner well from my days in Vegas working at UNLV (1998–1999, 2003–2004). This question might take too long on a 5 minute quiz just to read it out when the students are only half awake and frazzled. Just remember that displacements are vectors, and so you need to provide a direction.

Wrong answers:

a) One mile is the magnitude only of the displacement.
c) This would put you north of Cashman Field I think.
d) Nine miles isn't even the magnitude of the displacement.
e) You are not at the Hard Rock Cafe which isn't even on the Strip: its on Harmon East. (But doesn't it have a strip entrance?) But you didn't need to know that to know the answer is wrong.

Redaction: Jeffery, 2001jan01

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003 qmult 10810 3 3 2 tough math: hurricane fullmult

15.* The eye of a hurricane passes over Bermuda moving 10.0° north of west at 50 km/h for 3 hours and then turns due north moving at 30 km/h. What is its displacement relative to Bermuda after 6 hours from being at Bermuda: give distance from Bermuda and angle relative to north? Neglect the curvature of the Earth.

a) 101 km at 48° west of north.
b) 188 km at 52° west of north.
c) 157 km at 69° west of north.
d) 139 km at 31° west of north.
e) 202 km at 81° west of north.

SUGGESTED ANSWER: (b)

Well the displacements are 150 km at 10.0° north of west and 90 km due north. There are various ways of the doing vector addition. The simplest is probably to start by decomposing the first displacement into west and north components. These are 147.721 km west and 26.047 km north. Then add

26.047 km and 90 km to get 116.047 km for the total northward component. The magnitude of the displacement from Bermuda is then

$$\sqrt{147.721^2 + 116.047^2} = 188\,\text{km}$$

and the angle west from north is

$$\tan^{-1}\left(\frac{147.721}{116.047}\right) = 52^\circ \ .$$

Fortran-95 Code

```
          print*
          v1=50.d0
          t1=3.d0
          theta1=10.d0
          r1=v1*t1
          x1=r1*cos(theta1/raddeg)
          y1=r1*sin(theta1/raddeg)
          v2=30.d0
          t2=6.d0
          y2=v2*(t2-t1)
          x=x1
          y=y1+y2
          r=sqrt(x**2+y**2)
          theta=atan(x/y)*raddeg  ! This is west of north.
          print*,'r1,x1,y1,y2,x,y,r,theta'
          print*,r1,x1,y1,y2,x,y,r,theta
    !  150.0 147.7211629518312 26.04722665003955 90.0
147.7211629518312
    ! 116.04722665003955 187.85233774698443 51.8474264563665
```

Wrong answers:

a) This is the answer to the full-answer version of this question.

Redaction: Jeffery, 2008jan01

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004 qmult 00210 2 1 1 mod. deducto-mem.: gravity is always downward on Earth
Extra keywords: physci

16. A ball is tossed into the air and falls to the ground some distance away. Consider its motion in the vertical direction only and neglect air drag.

a) The ball has a constant acceleration downward.
b) The ball first accelerates **UPWARD** on its rising path and then accelerates **DOWNWARD** on its falling path.
c) The ball first accelerates **DOWNWARD** on its rising path and then accelerates **UPWARD** on its falling path.
d) The ball does not accelerate at all.
e) The ball is always accelerating in the upward direction.

SUGGESTED ANSWER: (a)

An easy memory question. Acceleration due to gravity alone is always downward and is a nearly constant near the Earth's surface. The magnitude of acceleration due to gravity alone is the magnitude of the gravitational field. Near the Earth's surface the gravitational field magnitude has fiducial value $g = 9.8\,\mathrm{m/s^2}$.

Wrong answers:

Redaction: Jeffery, 2001jan01

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004 qmult 00360 2 3 1 moderate math: ground speed from Pyth. theorem

Extra keywords: physci

17. You are flying a plane. Air velocity (i.e., plane velocity relative to the air) is 40 mi/h due north. Wind velocity is 30 mi/h due west. What is the magnitude of ground velocity (i.e., the ground speed)?

a) $50\,\mathrm{mi/h}$. b) $-50\,\mathrm{mi/h}$. c) $40\,\mathrm{mi/h}$. d) $10\,\mathrm{mi/h}$. e) $2500\,\mathrm{mi/h}$.

SUGGESTED ANSWER: (a)

This question was a natural for my pilot students at Middle Tennessee State University (MTSU). The magnitude of the ground velocity can be found by the Pythagorean theorem in this case because the two velocity vectors to be added are at right angles to each other. Deduction should also give the answer. But more formally:

$$\vec{v}_{\rm air} = (0, 40) \qquad \text{and} \qquad \vec{v}_{\rm wind} = (30, 0)$$

Thus,

$$\vec{v}_{\rm ground} = (30, 40) \qquad \text{and} \qquad v_{\rm ground} = \sqrt{30^2 + 40^2} = 50\,\mathrm{mi/h}\ .$$

Wrong answers:

b) Magnitude or speed is never negative.

d) No. This speed is only possible if the plane direction were opposite the wind direction.

e) You forgot to take the square root. But no subsonic aircraft velocity and wind velocity can give you this speed.

Redaction: Jeffery, 2001jan01

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004 qmult 00412 1 4 2 easy deducto-memory: radians in a circle 2

Extra keywords: mathematical physics

18. "Let's play *Jeopardy*! For \$100, the answer is: 2π."

What is ____________, Alex?

a) the number of degrees in a circle b) the number of radians in a circle c) twice the number of radians in a circle d) twice the number of degrees in a circle e) cherry and blueberry

SUGGESTED ANSWER: (b)

Wrong answers:

e) Actually, I've never been a big pie fan myself.

Redaction: Jeffery, 2008jan01

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004 qmult 00554 1 5 3 easy thinking: centripetal acceleration formula 2
Extra keywords: physci

19. From dimensional analysis (i.e., by checking for correct dimensions or units) or otherwise, one identifies the formula for centripetal acceleration to be:

a) $\vec{a}_{\rm cen} = -v^2 r\hat{r}$. b) $\vec{a}_{\rm cen} = -v^2 r^2\hat{r}$. c) $\vec{a}_{\rm cen} = -\dfrac{v^2}{r}\hat{r}$. d) $\vec{a}_{\rm cen} = vr\hat{r}$.
e) $\vec{a}_{\rm cen} = vr^2\hat{r}$.

SUGGESTED ANSWER: (c)

Wrong answers:
a) Wrong units.

Redaction: Jeffery, 2001jan01

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004 qmult 10320 3 5 3 tough thinking: Amazon crossing 1 fullmult

20.** Deep in the Amazonian jungle you wish to cross a river livid with piranha and crocodiles. Let x be the coordinate **ALONG** the river and y the coordinate **PERPENDICULAR** to the river. The river width is $y_{\rm max} = 40\,{\rm m}$. You are going to paddle across (on an unstable rotting log—using your bare hands) and your paddling speed is $v_{\rm paddle} = 0.50\,{\rm m/s}$.

Now let's make the not-too-likely assumption that the river velocity in the x-direction is a linear function of y, the distance in the y-direction. Let

$$v_x(y) = v_{x\ \rm max}\frac{y}{y_{\rm max}} \ ,$$

where $y = 0$ at the starting shore and $v_{x\ \rm max} = 1.5\,{\rm m/s}$. How far downstream (i.e., in the x direction) do you move?

a) 20 m. b) 40 m. c) 60 m. d) 80 m. e) 100 m.

SUGGESTED ANSWER: (c)

First note that the crossing time must be

$$t = \frac{y_{\rm max}}{v_{\rm paddle}} = \frac{40}{0.50} = 80\,{\rm s} \ .$$

Now note that your y position as a function of time is

$$y = v_y t$$

which implies that your x velocity as a function of time is

$$v_x(y) = v_{x\ \rm max}\frac{v_y}{y_{\rm max}}t \ .$$

Thus, your x acceleration is a constant of value $v_{x\,\mathrm{max}}v_y/y_{\mathrm{max}}$. Applying the appropriate constant-acceleration kinematic equation, we find that the downstream distance is

$$x = \frac{1}{2}at^2 = \frac{1}{2}\frac{v_{x\,\mathrm{max}}v_y}{y_{\mathrm{max}}}t^2 = \frac{1}{2} \times \frac{1.5 \times 0.5}{40} \times 80^2 = 60\,\mathrm{m}\ .$$

Wrong answers:

a) Too small.

Fortran-95 Code

```
          vy=0.5d0
          ymax=40.d0
          vxmax=1.5
          t=ymax/vy
          x=.5d0*(vxmax*vy/ymax)*t**2
          print*,'t,x'
          print*,t,x
    !  80.0 60.0
```

Redaction: Jeffery, 2008jan01

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002 qfull 00442 2 3 0 moderate math: Merc-Jag kinematic eqn. problem
Extra keywords: Mercedes and Jaguar

21. At time zero, there is a car (a Jaguar) stopped at a red light and a Mercedes (that has just run the red) is moving at a constant 50.0 m/s and is 20.0 m ahead of the Jag. Also at time zero, the stoplight turns green and the Jag (driven by an atavistic categorical imperative) starts accelerating forward at $5.00\,\mathrm{m/s^2}$.

a) What is 50.0 m/s in miles per hour? Note the modern mile is defined to be exactly 1609.344 meters.

b) Given the conditions, must the Jag pass the Mercedes? Why? (If your answer is no, you can skip the rest of the question.)

c) Calculate the time to the pass?

d) Calculate the distance from the stoplight to the pass?

e) How fast is the Jag going at the pass in meters per second **AND** miles per hour?

SUGGESTED ANSWER:

a) The conversion factor from meters per second to miles per hour is

$$\left(\frac{1\,\mathrm{mi}}{1609\,\mathrm{m}}\right)\left(\frac{1}{1\,\mathrm{h}/3600\,\mathrm{s}}\right) = 2.237\,\frac{\mathrm{mi/h}}{\mathrm{m/s}} \approx 2.25\,\frac{\mathrm{mi/h}}{\mathrm{m/s}}\ ,$$

and so the Mercedes speed is

$$50\,\mathrm{m/s} \times 2.237\,\frac{\mathrm{mi/h}}{\mathrm{m/s}} = 112\,\mathrm{mi/h}\ .$$

The Mercedes is flying.

b) Yes. Since the Jag's acceleration is a positive constant, it's distance must increase as the square of time whereas the Mercedes' distance increases only linearly with time. Eventually the Jag's distance must outgrow that of the Mercedes.

Another way of looking at it (which is perhaps even more clear) is to note that the Jag's speed must grow greater than the Mercedes's speed in a finite time because it is accelerating at a constant rate. Once that happens with only finite distance between them, the Jag must overtake the Mercedes in a finite time.

c) There are two bodies moving with constant acceleration, and so two sets of kinematic equations are needed in general to describe the motion. However, we immediately see that we need only the two position equations

$$x_{\mathrm{Mer}} = vt + x_0$$

for the Mercedes and

$$x_{\mathrm{Jag}} = \frac{1}{2}at^2$$

for the Jag. Passing gives us a condition that allows us to solve for the time of passing: i.e., passing occurs when $x_{\rm Mer} = x_{\rm Jag}$. If we equate our equations, we obtain the quadratic equation

$$0 = \frac{1}{2}at^2 - vt - x_0 \ .$$

The quadratic function $f(t) = \frac{1}{2}at^2 - vt - x_0$ is obviously a parabola opening upward. There can be two, one, or no zeros (i.e., roots) of the quadratic.

If the discriminant of the quadratic equation (i.e., the $b^2 - 4ac$ term of the general solution) is positive, then there are two passes: larger one gives the Jag passing the Mercedes and the smaller one an unreal pre-initial time pass of the backward-moving Jag by the Mercedes. If the discriminant is zero, then there are no passes, but the Mercedes does catch up to the Jag for a moment: we know this case is already ruled out since the Mercedes at time zero is ahead. If the discriminant is negative, the Mercedes never even catches up to the Jag: again we know the Mercedes is ahead at time zero, and so this case is already ruled out too.

We conclude that the discriminant must be positive and there is only one realized pass.

The solution of the quadratic is

$$t = \frac{v \pm \sqrt{v^2 + 2ax_0}}{a} = \frac{50 \pm \sqrt{2700}}{5} \approx -0.4\,{\rm s} \qquad {\rm or} \qquad 20.4\,{\rm s}\ ,$$

where the positive answer is the actual time of the pass.

The negative solution corresponds to a unrealized pass that occurred before time zero. The equations don't know that the Mercedes started accelerating at time zero. They think they apply at all times. In their view, the Mercedes moved in negative infinity and was passed by the Jag in the negative time region. A displacement motion diagram for the two objects makes clear what happened. Of course, the Jag did pass the Mercedes before time zero, but the Mercedes was a rest: this was before it started accelerating.

If we had our thinking hats on, we might that tried an approximate solution. The quadratic equation

$$0 = \frac{1}{2}at^2 - vt - x_0$$

can be approximated by

$$0 = \frac{1}{2}at^2 - vt$$

if x_0 is assumed to be small compared to the other terms. The approximate equation has $t = 0$ as solution which is clearly not a real solution and is the approximate solution to the negative solution we found above. The other solution is

$$t = \frac{2v}{a} = \frac{2 \times 50}{5} = 20\,{\rm s}$$

which is an approximation to the real solution that is not bad. We can check that it is not bad by noting that with $t = 20\,\mathrm{s}$ we get

$$\frac{1}{2}at^2 = 1000 >> x_0 = 20 \qquad \text{and} \qquad vt = 1000 >> x_0 = 20 \ .$$

So neglecting x_0 was a valid approximation.

d) The distance at the pass is given by

$$x_{\text{Jag}} = \frac{1}{2}at^2 = 1040\,\mathrm{m} \ .$$

e) The velocity of the Jag at the pass is given by

$$v_{\text{Jag}} = at = 102\,\mathrm{m/s} = 229\,\mathrm{mi/h} \ ,$$

which I find incredible for any ordinary commercial car—but then I've never driven on the German Autobahn, where velocities of this size are approached.

Fortran-95 Code

```
          print*
          v=50.d0
          x0=20.d0
          a=5.d0
          conv=(1.d0/1609.d0)*(3600.d0/1.d0)
          vmph=v*conv
          disc=v**2+2.d0*a*x0
          t1=(v-sqrt(disc))/a
          t2=(v+sqrt(disc))/a
          xpass=.5d0*a*t2**2
          vpass=a*t2
          vpassmph=vpass*conv
          print*,'conv,vmph,disc,t1,t2,xpass,vpass,vpassmph'
          print*,conv,vmph,disc,t1,t2,xpass,vpass,vpassmph
    ! 2.2374145431945304 111.87072715972653 2700.0
-0.39230484541326404
    ! 20.392304845413264 1039.6152422706632 101.96152422706632
228.13019715191964
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Redaction: Jeffery, 2001jan01

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004 qfull 00232 3 3 0 tough math: Lefty O'Doul homer

22. Lefty O'Doul—bats left, throws left—hits a home run that lands in the stands of Baker Bowl. The ball left the bat with initial horizontal velocity v_{x0} and inital vertical velocity v_{y0}. Neglect air drag.

a) What are the x and y coordinates of the ball as **FUNCTIONS** of time t during the ball's flight? Assume the initial position is the **ORIGIN** and that the ball flies off in the positive x direction.

b) Solve for v_{x0} and v_{y0} as functions of **ONLY** x, y, t and g.

c) Solve for the launch speed v_0 and launch angle θ from the horizontal as functions of **ONLY** x, y, t, and g. Note **ONLY** x, y, t, and g.

d) The ball lands 125 m in horizontal distance from the launch pad (i.e., homeplate) and 25 m above the launch height. The flight time is 3.98 s. What are launch speed and angle?

SUGGESTED ANSWER:

a) From the constant-acceleration kinematic equations, we find that

$$x = v_{x0}t \ ,$$
$$y = -\frac{1}{2}gt^2 + v_{y0}t \ .$$

b) Behold:

$$v_{x0} = \frac{x}{t} \ ,$$
$$v_{y0} = \frac{y + (1/2)gt^2}{t} \ .$$

c) Behold:

$$\begin{aligned} v_0 &= \frac{\sqrt{x^2 + [y + (1/2)gt^2]^2}}{t} \\ &= \sqrt{\left(\frac{x}{t}\right)^2 + \left(\frac{y}{t} + \frac{1}{2}gt\right)^2} \\ &= \sqrt{\left(\frac{x}{t}\right)^2 + \left(\frac{y}{t}\right)^2 + yg + \frac{1}{4}g^2t^2} \ , \\ \theta &= \tan^{-1}\left[\frac{y + (1/2)gt^2}{x}\right] \ . \end{aligned}$$

The first formula for v_0 is probably the most useful for calculations, but the other ones have some minor interest: at least they show that one can't find anything much better than the first one. Note that since we know that x is positive, we know we don't need to add 180° to the inverse tangent result to get the correct launch angle.

Note that one can't determine v_0 and θ uniquely from x and y alone. Just given location (x, y), there is a range of v_0 and θ values which will get the ball to that location. This is plausible just based on one's understanding of possible projectile trajectories. But the result is proven by our formulae for v_0 and θ. For fixed x and y, one can vary v_0 and θ by varying time.

d) Behold:

$$v_0 = \frac{\sqrt{x^2 + [y + (1/2)gt^2]^2}}{t} = 40.6\,\text{m/s} \ ,$$
$$\theta = \tan^{-1}\left[\frac{y + (1/2)gt^2}{x}\right] = 39^\circ \ .$$

By the by, Lefty in his greatest season, 1929, batted .398, hit 32 home runs, and had a record 254 base hits (Wikipedia: Lefty O'Doul). If he'd just hit .400, the "Man in the Green Suit" might have made it into the Hall of Fame.

Fortran-95 Code

```
      print*
      raddeg2=180.d0/acos(-1.d0)
      gg=9.8d0
      x=125.d0
      y=25.d0
      tt=3.98d0
      v0=sqrt(x**2+(y+.5d0*gg*tt**2)**2)/tt
      theta=atan((y+.5d0*gg*tt**2)/x)*raddeg2
      print*,'v0,theta'
      print*,v0,theta
!  40.63478727495055 39.38406778378693
```

Redaction: Jeffery, 2008jan01

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004 qfull 00310 2 3 0 moderate math: swimming English channel

23. You have decided to swim the English channel from Gris-Nez to Folkestone: a distance of about 40 km roughly to the north-west. In this problem, treat the distances and directions as exact: e.g., 40 km is the exact distance from Gris-Nez to Folkestone, north-west is exactly 45° west of north, and south-west is exactly 135° west of north. Also treat the Earth as flat. **HINT:** Smear your body all over with fats to prevent hypothermia.

a) If you swim at 4.0 km/h, how long will it take you?

b) As it turns out you land at New Romney about 20 km south-west of Folkestone. What is the average channel current velocity (note velocity, not just speed) during your swim? **HINT:** Velocity requires a direction specification too.

c) What was your average velocity (note velocity, not just speed) during the swim relative to the fixed Earth? As noted in the preamble, assume that Folkestone is exactly north-west of Gris-Nez and New Romney is exactly south-west of Folkestone. Give the velocity in magnitude-direction format. **HINT:** Velocity requires a direction specification too.

SUGGESTED ANSWER:

a) Behold:

$$t = \frac{d}{v} = \frac{40}{4.0} = 10\,\text{h} \ .$$

b) First note that since the unexpected displacment to the southwest was exactly perpendicular to the displacement that your velocity relative to the water would have given, the water must have been moving exactly perpendicular to velocity relative to the water. Therefore, the displacement to the southwest was entirely due to the water's motion.

With that established, one finds

$$v_{\text{current}} = \frac{d}{t} = \frac{20}{10} = 2\,\text{km/h} \ .$$

The direction is south-west: i.e., 135° west of north.

c) Well the two velocities, swimming and current, are exactly perpendicular (in the assumptions of this problem), and so the magnitude of the net velocity is

$$\sqrt{4^2 + 2^2} = \sqrt{20} \approx 4.5\,\text{km/h} \ .$$

The several ways of finding the direction. Probably, the simplest is just to take the inverse tangent function to find the angle θ that you actually traveled counterclockwise from due north-west. Behold:

$$\theta = \tan^{-1}\left(\frac{20}{40}\right) \approx 27^\circ \ .$$

Thus net velocity was at about 72° degrees west of north. A map actually suggests the direction would be more like 75° west of north. Given the crudity of the calculation anything in the vicinity of 70°–80° is OK.

Redaction: Jeffery, 2001jan01

SCRATCH PAGE

SCRATCH PAGE

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67428(67) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2006, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23),\mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

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4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1+x \ : \ (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1-\frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

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5 Quadratic Formula

If $\quad 0 = ax^2+bx+c\ ,\quad$ then $\quad x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = -\dfrac{b}{2a}\pm\sqrt{\left(\dfrac{b}{2a}\right)^2-\dfrac{c}{a}}$

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6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2+a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)+\pi? \qquad \vec{a}+\vec{b} = (a_x+b_x, a_y+b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2+a_y^2+a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right)+\pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

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7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0)$$
$$= f(x_0) + (x - x_0)f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

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8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\text{m/s}^2$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x \tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

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13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$