Intro Physics Semester I **Name:**

Homework 18: Thermodynamics I: Temperature, Heat, 1st Law One or two full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table

Name:

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

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019 qmult 00100 1 4 5 easy deducto-memory: thermodynamics defined 1

1. "Let's play *Jeopardy*! For $100, the answer is: The short definition is that it is the science of heat and temperature. The words heat and temperature require definitions of course."

   What is ____________, Alex?

   a) classical mechanics b) classical physics c) quantum mechanics d) thermostatics e) thermodynamics

   SUGGESTED ANSWER: (e)

   Wrong answers:

   b) Classical thermodynamics is a part of classical physics, but nowadays one almost never does pure classical thermodynamics. One is always mixing in statistical mechanics and quantum mechanics. It would be obtuse not too. You'd have to keep not saying what you know.

   Redaction: Jeffery, 2008jan01

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019 qmult 00110 1 1 3 easy memory: thermodynamics defined 2

2. In modern understanding, ____________ is the science of how microscopic structures control or affect certain macroscopic properties of the materials. These properties are described/measured by thermodynamic variables (also called state functions) which include internal energy, temperature, pressure, volume, density, and entropy. There are other variables. If none of its thermodynamic variables is changing, a system is in thermodynamic equilibrium (TE). At the microscopic level, atoms and molecules are changing state all the time in TE and out of TE. TE ____________ is much easier than non-TE ____________. Fortunately, many non-TE systems may be treated approximately as TE systems or can be understood as passing through a sequence of TE states. This extends the utility of TE ____________ into the non-TE realm. In this extension use is made of quasistatic processes. A quasistatic process is change in a system sufficient slow that the system can always be regarded as being in TE. To always exactly in TE, the quasistatic process would have to be infinitely slow. Fortunately, many not-so slow changes can be regarded as quasistatic process. Also imagining the infinitely slow quasistatic processes allows one to study limiting behaviors.

   a) classical mechanics b) classical physics c) thermodynamics d) quantum mechanics e) thermostatics

   SUGGESTED ANSWER: (c)

   Wrong answers:

   a) A nonsense answer.

   Redaction: Jeffery, 2008jan01

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019 qmult 00120 1 1 3 easy memory: internal energy defined

3. ____________ is the energy of microscopic structures that participate in thermodynamics. ____________ includes the kinetic energy of atoms, molecules and electrons relative to the rest frame of the bulk material. It also includes field energies for fields that vary on the atomic scale. The field energies can often be described as potential energies or as the electromagnetic radiation field energy. ____________ since it is an energy can be converted to or from macroscopic energies in various processes.

   a) in-to-it energy b) inner energy c) internal energy d) infernal energy e) gas energy

   SUGGESTED ANSWER: (c)

   Wrong answers:

   e) Gas energy could be understood as the internal energy of a gas.

   Redaction: Jeffery, 2008jan01

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019 qmult 00122 1 1 1 easy memory: heat and internal energy

4. Formally, ____________ (given the symbol Q) is internal energy transferred—but not transformed—by microscopic processes. In fact, almost everyone uses ____________ or ____________ energy as synonyms

for internal energy. Frequently, one says this is the ____________ transferred by a process and not as formally correct the ____________ of the process.

a) heat b) warmth c) calor d) frisson e) temperature

SUGGESTED ANSWER: (a)

Wrong answers:
c) In Spanish.

Redaction: Jeffery, 2008jan01

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019 qmult 00124 1 5 1 easy thinking: spontaneous heat flow
Extra keywords: physci

5. In the macroscopic world, heat spontaneously always flows from ____________. This result is common observation. In thermodynamics, it is understood as a consequence of the 2nd law of thermodynamics. We can make heat go the other way, but that takes work: it doesn't happen spontaneously.

a) hot to cold b) cold to hot c) hot to hotter d) hot to hottest e) objects at 0 K

SUGGESTED ANSWER: (a)

Wrong answers:
b) Exactly wrong.

Redaction: Jeffery, 2001jan01

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019 qmult 00130 1 1 2 easy memory: zeroth law of thermodynamics

6. If two systems each in TE are put into thermal contact (i.e., contact whereby internal energy can flow between) and there are no thermodynamic changes in either, then the two systems are said to be in TE with respect to each other whether they are in contact or not. If two systems are both in TE with respect to a third system, then it is an observational fact that they are in TE with respect to each other. This last fact is called the ____________ of thermodynamics. Why that name? Well it was decided that it was logically needed for thermodynamics after the first 3 laws had been well established. It could have been the called the 4th law, but it seemed more elementary than the other 3 laws. So the ____________ seems a happy choice.

a) 5th law b) zeroth law c) half law d) 1.1th law e) the old 100th law

SUGGESTED ANSWER: (b)

Wrong answers:
c) The half law?

Redaction: Jeffery, 2008jan01

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019 qmult 00140 1 4 4 easy deducto-memory: zeroth law and temperature

7. The existence of the zeroth law of thermodynamics suggests that there exists a thermodynamic variable that controls/describes the TE state of a system. It also suggests a way of measuring this variable for a system by changes in a small non-perturbing system as it comes into TE with the first system. The thermodynamic variable is an intensive variable: i.e., one that is independent of the size of a system. This must be since the condition of system being in TE with respect to each other is independent of their sizes. The TE state controlling/describing variable is called:

a) pressure. b) respiration. c) expiration. d) temperature. e) entropy.

SUGGESTED ANSWER: (d)

Wrong answers:
e) The 2nd law allows entropy to be defined.

Redaction: Jeffery, 2001jan01

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019 qmult 00150 1 4 2 easy deducto-memory: temperature defined

8. "Let's play *Jeopardy*! For \$100, the answer is: Well it's not the easiest thing to define. In modern understanding, it is thermodynamic variable that controls/describes the distribution of internal energy

among the microscopic states of a system. Its control is such that two systems with the same value of the parameter are in TE with respect to each other. The variable is an intensive variable since it is independent of the size of a system: it controls distribution, not amount, of energy. Unlike most intensive variables, the variable in question is not ratio of extensive quantities (those that do depend on the size of the sample) at least in any ordinary way of thinking of it. In a usual sense, we do cannot measure the variable in question directly. We measure some other quantity that is correlated with it over some range of behavior. For example, we it by measuring fluid volume in a fluid thermometer, gas pressure in a constant-volume gas thermometer, and electrical potential (i.e., voltage) with a thermocouple."

What is ____________, Alex?

a) heat   b) temperature   c) hotness   d) coldness   e) lukewarmness

SUGGESTED ANSWER: (b)

Wrong answers:
c) Well temperature is a quantitative measure of the human sense of warmness, coldness, and lukewarmness.

Redaction: Jeffery, 2008jan01

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019 qmult 00160 1 1 4 easy memory: 1st law of thermodynamics

9. The 1st law of thermodynamics is energy conservation in the field of the thermodynamics. There are various ways of writing it depending on circumstances. However, a conventional form for beginners is

$$dE = dQ - dW \ ,$$

where E is the internal energy of a system, Q is heat added to the system by microscopic heat transfer processes, and W is work (i.e., macroscopic work) done by the system. This formula for the 2nd law is written in terms of differentials since that is convenient in developing the formalism of thermodynamics and in problem-solving as it turns out. All the terms in the formula can be positive or negative. The minus sign in formula is annoying, but by convention one counts work done by a system (which takes away energy from it) rather than work done on the system (which adds energy to it). The 1st embodies energy conservation: contributions to a system's energy must change the system's energy.

The formula for differential work is ____________, where the p stands for system pressure and the V for system volume. This kind of work is given the name that comes from the vocalization of the letters in order followed by the word work.

a) $dW = p/V$ b) $dW = pV$ c) $dW = V\,dp$ d) $dW = p\,dV$ e) $dW = dp/V$

SUGGESTED ANSWER: (d)

Wrong answers:
a) A nonsense answer.

Redaction: Jeffery, 2008jan01

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019 qmult 00170 1 4 3 easy deducto-memory: pressure defined

10. "Let's play *Jeopardy*! For $100, the answer is: It is the thermodynamic variable that is the magnitude of the force per unit area that a material exerts on any internal or external surface. We usually think of it as being isotropic: i.e., having the same value for all directions at a point in the material. It can be anisotropic in some case."

What is ____________, Alex?

a) temperature   b) internal energy   c) pressure   d) density   e) entropy

SUGGESTED ANSWER: (c)

Wrong answers:
a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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019 qmult 00180 1 1 5 easy memory: derivation of pdV work

11. Actually, the formula for $p\,dV$ work requires a derivation. Consider a vectorized differential surface element $d\vec{A}$ at the surface of a system. The vector points in the perpendicular outward direction from the differential surface element itself and is well defined since a differential surface element is planar. The system pressure does work on the differential surface element if the surface element moves. We find

$$dW = p\,d\vec{A}\cdot d\vec{s} = p\,(dA\,ds\cos\theta)\ ,$$

where $d\vec{s}$ is the displacement of the surface element. A little thought shows that $(dA\,ds\cos\theta) = dV$ the change in volume with the displacement. Reflecting on the area of a rectangle and drawing a diagram might help. Thus, we have

$$dW = p\,dV$$

for the work done by the system. We note that $p\,dV$ work can be:

a) positive only. b) negative only. c) zero only. d) a square root.
e) positive or negative.

SUGGESTED ANSWER: (e) Being zero is an implied allowed case.

Wrong answers:
d) A nonsense answer.

Redaction: Jeffery, 2008jan01

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019 qmult 00190 1 1 2 easy memory: isothermal and adiabatic

12. A thermodynamic process at constant temperature is called ____________ and that with no heat flow (i.e., $dQ = 0$) is called ____________.

a) adiabatic; isothermal b) isothermal; adiabatic c) isobaric; isothermal
d) adiabatic; isobaric e) isobaric; adiabatic

SUGGESTED ANSWER: (b)

Wrong answers:
c) An isobaric process is one at constant pressure.

Redaction: Jeffery, 2008jan01

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019 qmult 00192 1 4 1 easy deducto-memory: ideal thermal processes

13. Isothermal, adiabatic, engine cycle, and free expansion processes: briefly and in order these processes can be characterized by the phrases:

a) (i) constant temperature, (ii) no heat flows (no entropy change), (iii) final state is the initial state and **NET** heat and work flow, and (iv) no heat flow and no work and not quasistatic.
b) (i) constant temperature, (ii) final state is the initial state and **ZERO NET** heat and work flow, (iii) no heat flows (no entropy change), and (iv) no heat flow and no work and not quasistatic.
c) (i) constant temperature, (ii) no heat flow and no work and not quasistatic, (iii) final state is the initial state and **ZERO NET** heat and work flow, and (iv) no heat flows (no entropy change).
d) (i) constant pressure, (ii) constant volume, (iii) constant entropy, and (iv) constant free energy.
e) (i) resonance frequency, (ii) first harmonic, (iii) Doppler effect, and (iv) Manifest Destiny.

SUGGESTED ANSWER: (a) Only the free expansion is necessarily not quasistatic: the others may be not quasistatic—practical engines usually are not quasistatic in fact.

Wrong answers:
e) Free expansion sounds like a 19th century election slogan to me.

Redaction: Jeffery, 2001jan01

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019 qmult 00200 1 1 1 easy memory: 3 temperature scales
Extra keywords: physci

14. The three common temperature scales are:

a) Fahrenheit, Celsius, and Kelvin. b) Fahrenheit, Celsius, and Newton.
c) Fahrenheit, Vesuvius, and Kelvin. d) Fahrenheit, Celsius, and Calvin.
e) Gesundheit, Vesuvius, and Calvin.

SUGGESTED ANSWER: (a)

Wrong answers:
e) Everyone one remembers *Calvin and Hobbes*: that great old comic strip—don't they?—is it later than I think.

Redaction: Jeffery, 2001jan01

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019 qmult 00202 1 4 2 easy deducto-memory: kelvin/absolute temperature
Extra keywords: physci
15. "Let's play *Jeopardy*! For $100, the answer is: This temperature scale is considered to be the absolute temperature scale and its zero-point is absolute zero."

What is the ____________ scale, Alex?

a) Fahrenheit b) Kelvin c) Celsius d) thermometer e) Hobbes

SUGGESTED ANSWER: (b)

Wrong answers:
a) Yep, good old Fahrenheit.

Redaction: Jeffery, 2001jan01

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019 qmult 00220 1 4 3 easy deducto-memory: triple point of water
16. "Let's play *Jeopardy*! For $100, the answer is: The name for the state in which water coexists in its three phases (vapor, liquid, and solid) at Kelvin temperature 273.16 K (by definition) and pressure 610 Pa."

What is ____________, Alex?

a) the unique point of water b) the water plasma phase c) the triple point of water d) the state where the Celsius and Fahrenheit temperature scales agree e) is hockey weather

SUGGESTED ANSWER: (c)

Wrong answers:
b) I don't think there can be a water plasma phase. In physics, as opposed to biology, a plasma is an ionized gas. If water were hot enough to be an ionized gas, I rather imagine that the water molecules would have been disassociated into hydrogen and oxygen.
d) This is -40°C which is also -40°F: either way, it's mighty nippy.
e) I would think the pressure was a little low—standard air pressure is about 10^5 Pa) and the ice a little too unsolid (if it coexists with water) for hockey.

Redaction: Jeffery, 2001jan01

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019 qmult 00240 1 1 1 easy memory: temperature conversion rules
17. The three main temperature scales (Kelvin, Celsius, and Fahrenheit) are related by following conversion rules

$$T_{\rm K} = T_{\rm C} + 273.15\,{\rm K} \ , \qquad T_{\rm F} = 1.8 \times T_{\rm C} + 32^\circ{\rm F} \ .$$

What are 290 K, 300 K, and 310 K in Fahrenheit?

a) 62, 80, 98. b) 59, 73, 96. c) 57, 71, 95. d) 56, 69, 94. e) 49, 64, 91.

SUGGESTED ANSWER: (a)

Well one uses

$$T_{\rm F} = 1.8 \times (T_{\rm K} - 273.15) + 32^\circ{\rm F}$$

in some kind of programming unit to calculate the values.

Now we can use Kelvin for all human relevant temperatures.

Wrong answers:
a) A nonsense answer.

Fortran-95 Code

```
print*
```

```
        do xk=290.d0,310.01d0,10.d0
           xc=xk-273.15d0
           xf=xc*1.8d0+32.d0
           print*,xk,xc,xf
        end do
!
!     290.00000000000000        16.850000000000023        62.330000000000041
!     300.00000000000000        26.850000000000023        80.330000000000041
!     310.00000000000000        36.850000000000023        98.330000000000041
```

Redaction: Jeffery, 2008jan01

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019 qmult 00242 1 3 2 easy math: temperature conversion example
Extra keywords: physci

18. What are 10° C, 20° C, and 30° C in Fahrenheit? **HINT:** I always multiply a Celsius temperature by 1.8 and add 32° to get Fahrenheit. Going the other way subtract 32° and divide by 1.8.

    a) −273.15° F, 273.15° F, 373.15° F. b) 50° F, 68° F, 86° F. c) 40° F, 48° F, 56° F. d) 48° F, 68° F, 90° F. e) 0° F, 100° F, 212° F.

SUGGESTED ANSWER: (b)

An easy math question. Super easy with the hint, but I think they really need that rule. Personally I think we all ought to use Kelvin all the time. It's real absolute temperature. Now it's awkward to say 288 K and the like for ordinary human uses. So as a convention for human activities make the leading 2 understood. Thus '88 K, etc where the apostrophe isn't spoken ust understood and written. But along with exponents of rational calendrical reform and Esperanto, I'll just have to live with perpetually dashed hopes. There is some deep-seated fixation on clunky systems in human nature.

Wrong answers:
e) Not likely.

Redaction: Jeffery, 2001jan01

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019 qmult 00244 1 3 1 easy math: boiling ethyl alcohol
Extra keywords: physci KB-130-23

19. Ethyl alcohol boils at about 172°F (at 1 atmosphere pressure one assumes). The conversion formula from Fahrenheit to Celsius is

$$T_{\rm C}=\frac{T_{\rm F}-32}{9/5} \ .$$

The boiling point on the Celsius scale is:

a) 77.8°C. b) 32°C. c) 0°C. d) 100°C. e) 172°C.

SUGGESTED ANSWER: (a)

Behold:

$$T_{\rm C}=\frac{T_{\rm F}-32}{9/5}=\frac{140}{9/5}=\frac{700}{9}\approx 77.8^\circ{\rm C} \ .$$

Fortran Code

```
        print*
        tf=172.
        tc=(tf-32)/1.8
        print*,'tc'
        print*,tc
*         77.77778
```

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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019 qmult 00252 1 1 4 easy memory: -40 same in celsius and fahrenheit

20. There is one temperature where the Celsius and Fahrenheit scales give the same number. What is that number?

   a) 40. b) 0. c) 32. d) -40. e) -32.

   SUGGESTED ANSWER: (d)

   We use conversion equation

$$T_{\rm F} = 1.8 \times T_{\rm C} + 32^\circ{\rm F}$$

   with $T_{\rm F} = T_{\rm C}$ and solve for $T_{\rm C}$. We get

$$T_{\rm C} = \frac{32}{1-1.8} = -40 \ .$$

   On either scale, it is pretty nippy.

   Wrong answers:
   a) A nonsense answer.

   Redaction: Jeffery, 2008jan01

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019 qmult 00600 1 4 4 easy deducto-memory: phase change
Extra keywords: physci

21. During a phase change of the common kind, the temperature of a substance in self-equilibrium (so all the material has the same temperature) is:

   a) 273.15 K. b) 32°F. c) changed by 10 % or more. d) constant. e) infinite.

   SUGGESTED ANSWER: (d) People probably remember this from somewhere.

   Wrong answers:
   a) This is the melting temperature of water and this would be the right answer for water, but the question is general.
   b) Same answer as (a).
   e) Oh, c'mon.

   Redaction: Jeffery, 2001jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \ : \ (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

$$\text{If} \qquad 0 = ax^2 + bx + c\ , \qquad \text{then} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series
$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\mathrm{m/s^2}$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x \tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\, dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F}\cdot d\vec{s} \qquad W = \int \vec{F}\cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F}\cdot\vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

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16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total } f} = KE_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

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17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\rm net} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\rm net} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho \, dV \qquad I_{\text{parallel axis}} = I_{\rm cm} + mR_{\rm cm}^2 \qquad I_z = I_x + I_y$$

$$I_{\rm cyl,shell,thin} = MR^2 \qquad I_{\rm cyl} = \frac{1}{2}MR^2 \qquad I_{\rm cyl,shell,thick} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\rm rod,thin,cm} = \frac{1}{12}ML^2 \qquad I_{\rm sph,solid} = \frac{2}{5}MR^2 \qquad I_{\rm sph,shell,thin} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$KE_{\rm rot} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z \, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta KE_{\rm rot} = W_{\rm net} = \int \tau_{z,\rm net} \, d\theta \qquad \Delta PE_{\rm rot} = -W = -\int \tau_{z,\rm con} \, d\theta$$

$$\Delta E_{\rm rot} = KE_{\rm rot} + \Delta PE_{\rm rot} = W_{\rm non,rot} \qquad \Delta E = \Delta KE + KE_{\rm rot} + \Delta PE = W_{\rm non} + W_{\rm rot}$$

19 Static Equilibrium

$$\vec{F}_{\rm ext,net} = 0 \qquad \vec{\tau}_{\rm ext,net} = 0 \qquad \vec{\tau}_{\rm ext,net} = \tau'_{\rm ext,net} \quad \text{if } F_{\rm ext,net} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\rm net} = \sum \tau$$

20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\rm escape} = \sqrt{\frac{2GM}{r}} \qquad v_{\rm orbit} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736 \times 10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5 \times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^{8} \approx 109 \times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g (y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}} g = V_{\text{fluid dis}} \rho_{\text{fluid}} g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2}\frac{d^2 y}{dt^2} \qquad v = \sqrt{\frac{F_{\text{T}}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_{\text{max}}\sin[k(x \mp vt)] = y_{\text{max}}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\text{max}}^2$$

$$y = 2y_{\max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \ : \ n = 1, 3, 5, \ldots \qquad f_{\mathrm{medium}} = \frac{f_0}{1 - v_0/v_{\mathrm{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

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24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K} \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32^{\circ}\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_{\mathrm{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\mathrm{RMS}}^2\right)$$

$$v_{\mathrm{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^{\gamma} = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\mathrm{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\mathrm{H}}} = \frac{Q_{\mathrm{H}} - Q_{\mathrm{C}}}{W} = 1 - \frac{Q_{\mathrm{C}}}{Q_{\mathrm{H}}} \qquad \eta_{\mathrm{heating}} = \frac{Q_{\mathrm{H}}}{W} = \frac{Q_{\mathrm{H}}}{Q_{\mathrm{H}} - Q_{\mathrm{C}}} = \frac{1}{1 - Q_{\mathrm{C}}/Q_{\mathrm{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\mathrm{cooling}} = \frac{Q_{\mathrm{C}}}{W} = \frac{Q_{\mathrm{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\mathrm{heating}} - 1$$

$$\varepsilon_{\mathrm{Carnot}} = 1 - \frac{T_{\mathrm{C}}}{T_{\mathrm{H}}} \qquad \eta_{\mathrm{heating,Carnot}} = \frac{1}{1 - T_{\mathrm{C}}/T_{\mathrm{H}}} \qquad \eta_{\mathrm{cooling,Carnot}} = \frac{T_{\mathrm{C}}/T_{\mathrm{H}}}{1 - T_{\mathrm{C}}/T_{\mathrm{H}}}$$