

Intro Physics Semester I**Name:**

Homework 17: Waves II: Sound One or two full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table**Name:**

	a	b	c	d	e		a	b	c	d	e
1.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	31.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	32.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	33.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	34.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
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6.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	36.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	37.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	38.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	39.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
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11.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	41.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
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14.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	44.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
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16.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	46.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
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23.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	53.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
24.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	54.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
25.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	55.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
26.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	56.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
27.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	57.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
28.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	58.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
29.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	59.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
30.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	60.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

1. The Doppler effect is a shift in wave frequency depending on the motion of source and/or observer. It is common to all wave phenomena, but the actual formulae vary from case to case. A prominent difference is the one between the sound Doppler effect and the light Doppler effect. Here we derive the formulae for the sound Doppler effect. We are going to stick to 1-dimensional cases for simplicity.

- a) Draw a schematic diagram of transverse wave cycle traveling in the positive x direction with wavelength λ and phase velocity v'' (not v' since we need that symbol for something else below). The wave cycle takes time P' to pass a given x position in the reference frame in which v'' is observed. Derive the formula

$$f'\lambda = v'' ,$$

where f' is the frequency of the wave.

- b) The formula $f'\lambda = v''$ derived in part (a) is obviously valid for a general frame of reference moving in the x direction. We will not consider frames moving in other directions. They are not so hard to deal with, but we want to keep this problem from getting too intricate. We are now in a position to see how f' is related to f the frequency of the wave in the medium's frame: i.e., the frame in which the medium is at rest.

First, note that the phase velocity v of the wave in medium's frame is the phase velocity that is derived from the medium properties and is just called the phase velocity without qualifications unless context says otherwise. The phase velocity in a general frame moving at velocity v' relative to the medium is

$$v'' = v - v' .$$

Second note that the wavelength λ of the wave is the same in all frames. Length is a frame invariant quantity in classical physics. This is not the case in relativistic physics which is needed for high relative velocities, but we are not going to consider those cases.

Now derive the basic sound Doppler shift formulae

$$f' = f \left(1 - \frac{v'}{v} \right) , \quad f = \frac{f'}{1 - v'/v} .$$

- c) The frequency f' of formula

$$f' = f \left(1 - \frac{v'}{v} \right)$$

can be interpreted as the frequency observed in a frame moving at v' for a fixed f . Draw a plot of f' versus v' and explain what is happening in the cases $v < 0$, $v' = 0$, $0 < v' < v$, $v = v'$, and $v' > v$. Assume $f > 0$ and $v > 0$ as usual.

- d) The frequency f formula

$$f = \frac{f'}{1 - v'/v}$$

can be interpreted as the frequency observed in the medium frame for a source moving at v' with a fixed source frequency f' . Draw a plot of f versus v' and explain what is happening in the cases $v < 0$, $v' = 0$, $0 < v' < v$, $v = v'$, and $v' > v$. Assume $f' > 0$ and $v > 0$ as usual.

- e) Say a source is moving at velocity v'_1 and emitting at fixed frequency f'_1 and an observer is moving at velocity v'_2 . Derive the formula for f'_2 in terms of v , v'_1 , v'_2 , and f'_1 .
- f) Taylor expand the result of part (e) to 1st order in small v_1/v and v_2/v , and derive the 1st order formula for

$$\frac{f'_2 - f'_1}{f'_1} .$$

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 2.998 \times 10^8 \text{ m/s} \approx 3 \times 10^8 \text{ m/s} \approx 1 \text{ yr/yr} \approx 1 \text{ ft/ns} \quad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19} \text{ C}$$

$$G = 6.67384(80) \times 10^{-11} \text{ N m}^2/\text{kg}^2 \quad (2012, \text{CODATA})$$

$$g = 9.8 \text{ m/s}^2 \quad \text{fiducial value}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.987551787 \dots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10} \text{ N m}^2/\text{C}^2 \text{ exact by definition}$$

$$k_{\text{Boltzmann}} = 1.3806504(24) \times 10^{-23} \text{ J/K} = 0.8617343(15) \times 10^{-4} \text{ eV/K} \approx 10^{-4} \text{ eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31} \text{ kg} = 0.510998910(13) \text{ MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27} \text{ kg} = 938.272013(23), \text{ MeV}$$

$$\epsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176 \dots \times 10^{-12} \text{ C}^2/(\text{N m}^2) \approx 10^{-11} \quad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \quad \text{exact by definition}$$

2 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \quad A_{\text{cir}} = \pi r^2 \quad A_{\text{sph}} = 4\pi r^2 \quad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\text{sphere}} = 4\pi \quad d\Omega = \sin\theta \, d\theta \, d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \quad \frac{y}{r} = \sin\theta \quad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \quad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \quad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \quad \cos(\theta + 180^\circ) = -\cos(\theta) \quad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \quad \cos(-\theta) = \cos(\theta) \quad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \quad \cos(\theta + 90^\circ) = -\sin(\theta) \quad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \quad \cos(180^\circ - \theta) = -\cos(\theta) \quad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \quad \cos(90^\circ - \theta) = \sin(\theta) \quad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \quad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \quad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}[\cos(a - b) - \cos(a + b)] \quad \cos(a)\cos(b) = \frac{1}{2}[\cos(a - b) + \cos(a + b)]$$

$$\sin(a)\cos(b) = \frac{1}{2}[\sin(a - b) + \sin(a + b)]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \quad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \quad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \quad \frac{1}{1-x} \approx 1+x : (x \ll 1)$$

$$\sin\theta \approx \theta \quad \tan\theta \approx \theta \quad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \quad \text{all for } \theta \ll 1$$

5 Quadratic Formula

$$\text{If } 0 = ax^2 + bx + c, \quad \text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \quad \theta = \tan^{-1} \left(\frac{a_y}{a_x} \right) + \pi? \quad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad \phi = \tan^{-1} \left(\frac{a_y}{a_x} \right) + \pi? \quad \theta = \cos^{-1} \left(\frac{a_z}{a} \right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab \cos \theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab \sin(\theta) \hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \quad \frac{d(x^0)}{dx} = 0 \quad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0)$$

$$= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!}f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!}f^{(3)}(x_0) + \dots$$

$$\int_a^b f(x) dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \quad \int \frac{1}{x} dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \quad v = \frac{dx}{dt} \quad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \quad x = \frac{1}{2}at^2 + v_0t + x_0 \quad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \quad x = -\frac{1}{2}at^2 + vt + x_0 \quad g = 9.8 \text{ m/s}^2$$

$$x_{\text{rel}} = x_2 - x_1 \quad v_{\text{rel}} = v_2 - v_1 \quad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}}t \quad v' = v - v_{\text{frame}} \quad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \quad \vec{v} = \frac{d\vec{r}}{dt} \quad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \quad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \quad v_{x,0} = v_0 \cos \theta \quad v_{y,0} = v_0 \sin \theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos \theta} \quad y = y_0 + x \tan \theta - \frac{x^2 g}{2v_0^2 \cos^2 \theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin \theta \cos \theta}{g} \quad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2 \theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2 \sin \theta \cos \theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \quad \theta_{\text{for max}} = \frac{\pi}{4} \quad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \quad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \quad \vec{v} = \vec{v}_2 - \vec{v}_1 \quad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \quad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \quad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \quad v = r\omega \quad a_{\text{tan}} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \quad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \quad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \quad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r}) \vec{r} dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \quad \vec{F}_{21} = -\vec{F}_{12} \quad F_g = mg \quad g = 9.8 \text{ m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \quad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \quad T = T_0 - F_{\text{parallel}}(s) \quad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \quad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \quad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r \frac{d\theta}{dt} \quad a_{\text{tangential}} = r\alpha = r \frac{d\omega}{dt} = r \frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r} \hat{r} \quad \vec{F}_{\text{centripetal}} = -m \frac{v^2}{r} \hat{r}$$

$$F_{\text{drag, lin}} = bv \quad v_{\text{T}} = \frac{mg}{b} \quad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \quad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag, quad}} = bv^2 = \frac{1}{2} C \rho A v^2 \quad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \quad W = \int \vec{F} \cdot d\vec{s} \quad KE = \frac{1}{2}mv^2 \quad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \quad P = \frac{dW}{dt} \quad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \quad \vec{F} = -\nabla PE \quad PE = \frac{1}{2}kx^2 \quad PE = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \quad \Delta KE_{\text{cm}} = W_{\text{net,external}} \quad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \quad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \quad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}}) \frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}} \frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \quad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t) dt \quad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \quad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \quad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total } f} = KE_{\text{total } i} \quad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \quad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \quad v_{\text{rel}'} = -v_{\text{rel}} \quad \text{1-d Elastic Collision Expressions}$$

17 Rotational Kinematics

$$2\pi = 6.2831853\dots \quad \frac{1}{2\pi} = 0.15915494\dots$$

$$\frac{180^\circ}{\pi} = 57.295779\dots \approx 60^\circ \quad \frac{\pi}{180^\circ} = 0.017453292\dots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \quad \omega = \frac{d\theta}{dt} = \frac{v}{r} \quad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \quad f = \frac{\omega}{2\pi} \quad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \quad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \quad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \quad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \quad \vec{\tau} = \vec{r} \times \vec{F} \quad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin \gamma_L \quad \tau_z = RF_{xy} \sin \gamma_\tau \quad L_z = I\omega \quad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \quad I = \int R^2 \rho dV \quad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \quad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \quad I_{\text{cyl}} = \frac{1}{2}MR^2 \quad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \quad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \quad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g \sin \theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \quad dW = \tau_z d\theta \quad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}} d\theta \quad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}} d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \quad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \quad \vec{\tau}_{\text{ext,net}} = 0 \quad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \quad 0 = F_{\text{net } y} = \sum F_y \quad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1 m_2}{r_{12}^2} \hat{r}_{12} \quad \vec{g} = -\frac{GM}{r^2} \hat{r} \quad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1 m_2}{r_{12}} \quad V = -\frac{GM}{r} \quad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \quad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \quad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2} \quad \frac{dA}{dt} = \frac{1}{2} r^2 \omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0 \text{ km} \quad R_{\text{Earth,equatorial}} = 6378.1 \text{ km} \quad M_{\text{Earth}} = 5.9736 \times 10^{24} \text{ kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11} \text{ m} = 1.0000001124 \text{ AU} \approx 1.5 \times 10^{11} \text{ m} \approx 1 \text{ AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^8 \approx 109 \times R_{\text{Earth,equatorial}} \quad M_{\text{Sun}} = 1.9891 \times 10^{30} \text{ kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \quad p = \frac{F}{A} \quad p = p_0 + \rho g d_{\text{depth}}$$

$$\text{Pascal's principle} \quad p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \quad \Delta p = \Delta p_{\text{ext}}$$

$$\text{Archimedes principle} \quad F_{\text{buoy}} = m_{\text{fluid dis}} g = V_{\text{fluid dis}} \rho_{\text{fluid}} g$$

$$\text{equation of continuity for ideal fluid} \quad R_V = Av = \text{Constant}$$

$$\text{Bernoulli's equation} \quad p + \frac{1}{2} \rho v^2 + \rho g y = \text{Constant}$$

22 Oscillation

$$P = f^{-1} \quad \omega = 2\pi f \quad F = -kx \quad PE = \frac{1}{2} kx^2 \quad a(t) = -\frac{k}{m} x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \quad P = 2\pi \sqrt{\frac{m}{k}} \quad x(t) = A \cos(\omega t) + B \sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} k x_{\text{max}}^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$P = 2\pi \sqrt{\frac{I}{mgr}} \quad P = 2\pi \sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2} \quad v = \sqrt{\frac{F_T}{\mu}} \quad y = f(x \mp vt)$$

$$y = y_{\text{max}} \sin[k(x \mp vt)] = y_{\text{max}} \sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \quad k = \frac{2\pi}{\lambda} \quad v = f\lambda = \frac{\omega}{k} \quad P \propto y_{\text{max}}^2$$

$$y = 2y_{\max} \sin(kx) \cos(\omega t) \quad n = \frac{L}{\lambda/2} \quad L = n \frac{\lambda}{2} \quad \lambda = \frac{2L}{n} \quad f = n \frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \quad n\lambda = d \sin(\theta) \quad \left(n + \frac{1}{2}\right)\lambda = d \sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \quad \beta = (10 \text{ dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n \frac{v}{4L} : n = 1, 3, 5, \dots \quad f_{\text{medium}} = \frac{f_0}{1 - v_0/v_{\text{medium}}}$$

$$f' = f \left(1 - \frac{v'}{v}\right) \quad f = \frac{f'}{1 - v'/v}$$