Intro Physics Semester I **Name:**

Homework 16: Waves I: One or two full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table

Name:

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

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017 qmult 00100 1 4 5 easy deducto-memory: waves defined

Extra keywords: physci KB-176

1. "Let's play *Jeopardy*! For \$100, the answer is: It is a extended-in-space location varying oscillation of something. Transport of energy and momentum can occur, but not in all cases."

   What is ____________, Alex?

   a) mass   b) energy   c) pressure   d) temperature   e) a wave phenomenon

   SUGGESTED ANSWER: (e)

   Wrong answers:
   b) Waves can transport energy.

   Redaction: Jeffery, 2001jan01

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017 qmult 00110 1 4 1 easy deducto-memory: 3 kinds of waves

Extra keywords: physci

2. In one method of physically classifying waves, one has:

   a) mechanical, electromagnetic, and quantum mechanical waves.
   b) left, right, and middle waves.
   c) S, P, and Middle Earth waves.
   d) American waves, British waves, and Australian wives.
   e) mechanical, financial, and emotional waves.

   SUGGESTED ANSWER: (a)

   Wrong answers:
   e) Not the best answer in the context of physics.

   Redaction: Jeffery, 2001jan01

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017 qmult 00120 1 4 5 easy deducto-memory: transverse and longitudinal waves

Extra keywords: physci

3. In another method of physically classifying waves, one has ____________ waves:

   a) trapeze and leotard   b) terrific and lewd   c) tornado and lounge
   d) toasty and lemony   e) transverse and longitudinal

   SUGGESTED ANSWER: (e)

   Wrong answers:
   d) Oh, c'mon.

   Redaction: Jeffery, 2001jan01

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017 qmult 00130 1 4 5 easy deducto-memory: basic wave equation

4. "Let's play *Jeopardy*! For \$100, the answer is:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} ,$$

   where the symbol ∂ indicates partial derivatives (i.e., $\partial y/\partial x$ is the derivative of y with respect to x holding t constant) y is the displacement of some wave quantity, x is a spatial dimensional, t is time, and v is a phase velocity. This equation and multi-dimensional generalization of it hold for many wave phenomena such as small waves on a string and electromagnetic waves. However, not all waves obey it."

   What is the ____________, Alex?

   a) medium   b) phase velocity equation   c) partial equation   d) phase equation
   e) wave equation

   SUGGESTED ANSWER: (e)

The wave equation is called the wave equation without qualification even though it is not the wave equation for everything. There are other wave equations. Maybe many others. But **THE** wave equation is very important turns up in many contexts. It is the electromagnetic wave equation which is an extremely important case. Maybe it should be called the basic wave equation—but people don't do that.

Wrong answers:
a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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017 qmult 00132 1 4 4 easy deducto-memory: linearity of the wave equation

5. What does the linearity of the wave equation (differential wave equation) imply?

   a) A linear combination of solutions is the square of a solution.
   b) A linear combination of solutions is the inverse of a solution.
   c) That there are no solutions.
   d) A linear combination of solutions is a solution.
   e) That there is only one solution.

SUGGESTED ANSWER: (d)

Only one solution seems leading and it's not a trick. Satisfying (d) is in fact the definition of linearity for a differential equation.

Wrong answers:
a) Nah.
b) Nah.
c) There are waves: seems unlikely unless all is wrong.
e) Nah.

Redaction: Jeffery, 2001jan01

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017 qmult 00134 1 1 4 easy memory: superposition principle

6. The ____________ principle for the wave equation—which is not the only wave equation despite its unqualified name—is not new axiom. It is is just a consequence of the linearity of the wave equation which implies that the linear combination of any two solutions is a solution. The ____________ principle applies in many context which is perhaps why it is gloried with the term "principle" even though it may not be an axiom in any of them as far as yours truly knows.

   a) wave   b) wavelength   c) symmetrization   d) superposition   e) werewolf

SUGGESTED ANSWER: (d)

Wrong answers:
c) This is an axiom in quantum mechancis

Redaction: Jeffery, 2008jan01

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017 qmult 00140 1 1 3 easy memory: basic wave equation general solution

7. A very general, but not completely general, solution of the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

is $f(x - vt)$. The proof for this solution is as follows. Let $\phi = x - vt$. Now

$$\text{LHS} = \frac{\partial^2 f}{\partial \phi^2} \times 1^2 = \frac{\partial^2 f}{\partial \phi^2} \;, \qquad \text{RHS} = \frac{1}{v^2}\frac{\partial^2 f}{\partial \phi^2} \times v^2 = \frac{\partial^2 f}{\partial \phi^2} = \text{LHS} \;,$$

and that is QED. We have made use of the:

   a) quotient rule.   b) product rule.   c) chain rule.   d) you-may-never-break-the-chain rule.
   e) right-hand rule.

SUGGESTED ANSWER: (c)

Wrong answers:
b) Tempting.

Redaction: Jeffery, 2008jan01

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017 qmult 00150 1 1 4 easy memory: general traveling wave solution

8. A very general, but not completely general, solution of the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

is $f(x - vt)$. The solution is a traveling wave solution. The proof for this solution is as follows. Let $\phi = x - vt$. For a constant ϕ, one has a constant oscillation deviation $f(\phi)$. But this deviation's x position moves as __________ with velocity __________. Since ϕ is called the phase of the wave solution, v is called the phase velocity. A wave solution of the form $f(x - vt)$ is just a pattern that is traveling to the right for $v > 0$ and to the left for $v < 0$. If $v^2 = 0$, the wave equation is not defined and there is no solution. But there can be solutions that do not travel.

a) $\phi - vt$; $-v$. b) $\phi + vt$; $-v$. c) $\phi - vt$; v. d) $\phi + vt$; v. e) v; $\phi - vt$.

SUGGESTED ANSWER: (d)

Since $\phi = x - vt$ is a constant, we find the location of the deviation $f(\phi)$ evolves according to $x = \phi + vt$.

Wrong answers:
a) As Lurch would say AAAAaarrrgh.

Redaction: Jeffery, 2008jan01

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017 qmult 00160 1 1 1 easy memory: standing waves

9. A non-traveling wave solution to the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

is

$$y = A\cos(kx + \phi_x)\cos(\omega t + \phi_t) ,$$

where A is amplitude, k is wavenumber t is angular frequency, and ϕ_x and ϕ_t are general phase constants. We require that $v = \omega/k$. Either or both of the cosine functions can be changed to sine functions by defining different phase constants: e.g., $\phi_x = \phi_x' - \pi/2$ which gives $\cos(kx + \phi_x) = \sin(kx + \phi_x')$. The proof for this solution is as follows: Now

$$\text{LHS} = \frac{\partial^2 y}{\partial x^2} = -k^2 y , \qquad \text{RHS} = \frac{1}{v^2}\frac{\partial^2 y}{\partial x^2} = -\frac{\omega^2}{v^2}y = -k^2 y = \text{LHS} ,$$

and that is QED. The solution does not travel. No shape simply glides to the right or left. At each point x, the solution is just simple harmonic motion with angular frequency ω and amplitude $A\cos(kx + \phi_x)$. Solutions of this kind are called:

a) standing waves. b) traveling waves. c) sitting waves. d) immobilized waves.
e) v; $\phi - vt$.

SUGGESTED ANSWER: (a)

Wrong answers:
b) Exactly wrong.

Redaction: Jeffery, 2008jan01

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017 qmult 00170 1 4 5 easy deducto-memory: waves transport energy
Extra keywords: physci KB-212-3

10. "Let's play *Jeopardy*! For $100, the answer is: Energy."

What ____________, Alex?

a) is vis viva b) is angular momentum c) is momentum d) is a vector
e) do waves transport

SUGGESTED ANSWER: (e)

Wrong answers:
a) Vis viva (Latin for living force) is an older name for kinetic energy. See Ca-96, 245, 295.

Redaction: Jeffery, 2001jan01

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017 qmult 00200 1 1 3 easy memory: wavelength and period defined
Extra keywords: physci KB-212-1

11. The **DISTANCE** along a wave pattern in its propagation direction before the shape begins to repeat is called the ____________ and the time period before the wave pattern begins to repeat itself at any point in space is called the ____________.

a) frequency f; epoch e b) amplitude A; duration d c) wavelength λ; period P
d) period P; aeon a e) phase velocity v; awhile a

SUGGESTED ANSWER: (c)

Wrong answers:
d) No. This is the time for one wavelength to pass a given point.

Redaction: Jeffery, 2001jan01

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017 qmult 00210 1 1 5 easy memory: frequency-period formula

12. Say N cyles (i.e., wavelengths) of a periodic wave have passed a given point. This took time NP. The number of cycles per unit time or frequency f is given by:

a) $f = P/N$. b) $f = 1/(NP)$. c) $f = N/P$. d) $f = P$ e) $f = 1/P$.

SUGGESTED ANSWER: (e)

Behold:

$$f = \frac{N}{NP} = \frac{1}{P} .$$

Wrong answers:
d) Exactly wrong.

Redaction: Jeffery, 2008jan01

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017 qmult 00220 1 4 2 easy deducto-memory: frequency-wavelength formula
Extra keywords: physci

13. The time for wave cycle of wavelength λ to pass a given point is period P. The phase speed of the wave phenomena is then ____________ from which one obtains the very familiar frequency-wavelength formula ____________.

a) $v = \lambda P$; $f = \lambda/v$ b) $v = \lambda/P$; $f\lambda = v$ c) $v = \lambda/P$; $\lambda = vf$ d) $v = \lambda/P$; $v^2 = f\lambda$
e) $v = \lambda P$; $\lambda^2 = vf$

SUGGESTED ANSWER: (b)

By inspection nearly only

$$v = \frac{\lambda}{P} = f\lambda .$$

Wrong answers:
a) Exactly wrong and the dimensions are wrong too.

Redaction: Jeffery, 2001jan01

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017 qmult 00230 1 1 1 easy memory: sinusoidal waves defined

14. ____________ waves in one dimesion are given by

$$y = A\cos(kx - \omega t + \phi) \qquad \text{or} \qquad y = B\cos(kx - \omega t) + C\sin(kx - \omega t) \ ,$$

where A is amplitude, k is wavenumber, x is positio, ω is angular frequency, t is time, and B are C are constants obtained by expanding $\cos(kx - \omega t + \phi)$ into cosine and sine terms. ____________ waves turn up in many contexts and, in fact, all waves can be expanded into linear combinations of them using Fourier series or Fourier transforms which gives these waves a universal use.

a) Sinusoidal b) Cosinusoidal c) Trigonometric d) Tangential e) Cotangential

SUGGESTED ANSWER: (a)

Wrong answers:
b) A unused expression.

Redaction: Jeffery, 2008jan01

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017 qmult 00232 1 1 4 easy memory: wavenumber wavelength, angular frequency frequency

15. A sinusoid repeat every time its argument increases by 2π. Thus, sinusoidal waves repeat as spatial coordinate alone varies when $k\Delta x = 2\pi$ and as time coordinate varies alone when $\omega\Delta t = 2\pi$ Immediately, one sees that:

a) $k = \lambda$ and $f = \omega$. b) $k = \pi\lambda$ and $f = \pi\omega$. c) $k = 2\pi\lambda$ and $f = 2\pi\omega$.
d) $k = 2\pi/\lambda$ and $f = \omega/(2\pi)$. e) $k = f$ and $f = \lambda$.

SUGGESTED ANSWER: (d)

Wrong answers:
a) This is the easy answer.
c) Exactly wrong.

Redaction: Jeffery, 2008jan01

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017 qmult 00236 1 4 2 easy deducto memory: sinusoidal periodic wave

16. A sinusoidal wave is an example of a/an:

a) aperiodic wave. b) periodic wave. c) transverse wave. d) longitudinal wave.
e) trapeze wave.

SUGGESTED ANSWER: (b)

Wrong answers:
a) Exactly wrong.
c) A sinusoidal wave can be either transverse or longitudinal.
d) A sinusoidal wave can be either transverse or longitudinal.

Redaction: Jeffery, 2001jan01

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017 qmult 00240 1 1 3 easy memory: sinusoidal waves and phase velocity

17. Sinusoidal waves, given by

$$y = A\cos(kx - \omega t + \phi) \qquad \text{or} \qquad y = B\cos(kx - \omega t) + C\sin(kx - \omega t) \ ,$$

satisfy the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \ .$$

This can be prove by direct subsitution into wave equation or by recognizing the sinusoidal wave function is of the form of the general traveling wave solution of the wave equation: i.e., $f(x - vt)$. Either way we find the relationship between phase velocity v and k and ω to be:

a) $v = \omega k^2 = f\lambda$. b) $v = \omega k = f\lambda$. c) $v = \omega/k = f\lambda$. d) $v = k/\omega = f\lambda$.
e) $v = \omega/k^2 = f\lambda$.

SUGGESTED ANSWER: (c)

Wrong answers:
b) Exactly wrong.

Redaction: Jeffery, 2008jan01

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017 qmult 00300 1 1 2 easy memory: wave equation for string

18. Consider a string running along the x axis with tension τ. We can derive equation of motion for small transverse waves on the string. The waves are small continuously varying displacements of the string in the transverse y direction. Consider a differential segment of the string that in the x direction extends length dx. Newton 2nd law applied to this segment of string gives

$$(\mu\,dx)\frac{d^2y}{dt^2} = (\tau\sin\theta)_2 - (\tau\sin\theta)_1 \ ,$$

where μ is the linear mass density, θ is the angle of the string from the vertical, $(\tau\sin\theta)_2$ is the vertical force at the right end of the segment, and $(\tau\sin\theta)_1$ is the vertical force at the left end of the segment. Since we are considering small waves, we can make the small-angle approximation and obtain

$$\sin\theta \approx \tan\theta = \frac{dy}{dx} \ .$$

We now assume that the tension τ can be approximated as constant despite the stretching of the string by the waves. This approximation is difficult to justify a priori, but the resulting equation of motion works very well in many cases and that gives a posteriori justification. Now we have

$$(\mu\,dx)\frac{d^2y}{dt^2} = \tau\,d(\tan\theta) = \tau\,d\left(\frac{dy}{dx}\right) \ ,$$

and thus

$$\frac{d^2y}{dt^2} = \frac{\tau}{\mu}\frac{dy^2}{dx^2} \ ,$$

is our equation of motion. Note the μ is constant if all the matter displacements are transverse. If they are not, we assume that μ can be approximated as constants. We recognize our equation of motion as the 1-dimensional wave equation with phase velocity ____________. From this recognition, all the formalism developed for the wave equation applies for waves on a string in the small-wave approximation. What if the waves are not small? Then the wave equation—which is not the only wave equation despite its unqualified name–does not apply and a more complex analysis of the string waves is needed.

a) $v = \dfrac{\tau}{\mu}$ b) $v = \sqrt{\dfrac{\tau}{\mu}}$ c) $v = \dfrac{\mu}{\tau}$ d) $v = \sqrt{\dfrac{\mu}{\tau}}$ e) $v = \sqrt{\mu\tau}$

SUGGESTED ANSWER: (b)

Wrong answers:
e) Oh, c'mon.

Redaction: Jeffery, 2008jan01

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017 qmult 00330 1 3 5 easy math: linear density

19. A string is 8.0 m long and has mass 0.020 kg. What is its linear density μ?

a) 400 kg/m. b) 250 kg/m. c) 400 m/kg. d) 2.5×10^{-5} kg/m. e) 2.5×10^{-3} kg/m.

SUGGESTED ANSWER: (e)

Behold:

$$\mu = \frac{m}{\ell} = \frac{0.020}{8.0} = 0.0025\,\mathrm{kg/m} \ .$$

Wrong answers:
c) Wrong units.

Redaction: Jeffery, 2001jan01

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017 qmult 00340 1 3 4 easy math: wavelength on a string

20. The wave speed for a string is

$$v = \sqrt{\frac{F_{\rm T}}{\mu}} \ ,$$

where $F_{\rm T}$ is the string tension and μ is the linear density (i.e., mass per unit length). What is wavelength as a function of $F_{\rm T}$?

a) $\lambda = f\sqrt{F_{\rm T}\mu}$. b) $\lambda = f\sqrt{F_{\rm T}/\mu}$. c) $\lambda = f^{-1}\sqrt{F_{\rm T}\mu}$. d) $\lambda = f^{-1}\sqrt{F_{\rm T}/\mu}$.
e) $\lambda = fF_{\rm T}\mu$.

SUGGESTED ANSWER: (d)

Wrong answers:

Redaction: Jeffery, 2001jan01

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017 qmult 00620 1 4 3 easy deducto-memory: what's an antinode?

21. What is an antinode?

a) A point of no motion in standing waves.
b) A point of minimum amplitude in standing waves.
c) A point of maximum amplitude in standing waves.
d) That which proceeds a node.
e) That which follows a node.

SUGGESTED ANSWER: (c)

Wrong answers:
a) That's a node.
b) Nah.
d) Sounds plausible.
e) Not even specious.

Redaction: Jeffery, 2001jan01

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017 qmult 00630 2 3 5 moderate math: standing wave wavelength on a string

22. You have a string of length L with fixed endpoints. There are standing waves on the string. You count n antinodes. What is the wavelength of the waves?

a) $\lambda = Ln$. b) $\lambda = L/n$. c) $\lambda = L^2/n$. d) $\lambda = L$. e) $\lambda = 2L/n$.

SUGGESTED ANSWER: (e)

Wrong answers:

Redaction: Jeffery, 2001jan01

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017 qmult 00640 1 4 3 easy deducto-memory: antinodes and humps

23. In the 3rd harmonic of standing waves on a string fixed at both ends, how many antinodes are there:

a) Six: 2 for each of the 3 full wavelengths making up the pattern.
b) Four: the endpoints and the 2 inner points of no motion.
c) Three.
d) Two like the Bactrian camel.
e) One like the Arabian camel.

SUGGESTED ANSWER: (c) It's just a fact, there an antinode for each half wavelength and the number of half wavelengths is the harmonic number.

Wrong answers:
b) Those are the nodes.
d) Not Old Joe Camel.
e) Maybe Old Joe Camel.

Redaction: Jeffery, 2001jan01

017 qfull 00330 1 3 0 easy math: a sinusoidal traveling wave

24. The equation of a transverse wave on a string is

$$y(t) = 9.0\sin(0.01\pi x + 4.0\pi t) \ ,$$

where x and y are in centimeters, t is in seconds, and the argument of the sine is in radians. Find the (a) amplitude A, (b) wavelength λ, (c) frequency f, (d) phase speed $v_{\rm ph}$, (e) direction of propagation, and (f) maximum transverse speed of the string. Also (g) what is the transverse displacement at $x = 3.5\,\text{cm}$ and time $t = 0.26\,\text{s}$?

SUGGESTED ANSWER:

a) $A = 9.0\,\text{cm}$

b) $\lambda = 2\pi/k = 2\pi/(0.01\pi) = 200\,\text{cm} = 2.0\,\text{m/s}$.

c) $f = \omega/(2\pi) = 2.0\,\text{Hz}$.

d) $v_{\rm ph} = -\omega/k = 400\,\text{cm/s} = -4.0\,\text{m/s}$. Note the fiducial argument of a traveling wave is $\phi = x - v_{\rm ph}t$. For constant ϕ, we must have x increase as given by

$$x = v_{\rm ph}t + \phi \ .$$

Thus, $v_{\rm ph} > 0$ gives travel to the right (positive velocity) and $v_{\rm ph} < 0$ gives travel to the left (negative velocity). For sinusoidal waves, $kx - \omega t = k[x - (\omega/k)t]$, and so we see that $v_{\rm ph} = -\omega/k$ which is what we have used.

e) The direction of propagation is in the negative x-direction since $v_{\rm ph} < 0$ as we established in the part (d) answer.

f) $v_{y,\,max} = \omega A = 36. \times \pi \approx 110\,\text{cm/s} \approx 1.1\,\text{m/s}$.

g) $y(x = 3.5, t = 0.26) \approx 9.0\sin(1.08\pi) \approx 9.0 \times (-0.08\pi) \approx 2.5\,\text{cm}$ to about 1-digit accuracy.

Redaction: Jeffery, 2001jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$
\begin{aligned}
c &= 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition} \\
e &= 1.602176487(40) \times 10^{-19}\,\mathrm{C} \\
G &= 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)} \\
g &= 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value} \\
k &= \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition} \\
k_{\mathrm{Boltzmann}} &= 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K} \\
m_e &= 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV} \\
m_p &= 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV} \\
\varepsilon_0 &= \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)} \\
\mu_0 &= 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}
\end{aligned}
$$

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2 Geometrical Formulae

$$
C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3
$$

$$
\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi
$$

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3 Trigonometry Formulae

$$
\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1
$$

$$
\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}
$$

$$
c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}
$$

$$
f(\theta) = f(\theta + 360^\circ)
$$

$$
\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)
$$

$$
\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)
$$

$$\sin(\theta+90^\circ)=\cos(\theta) \qquad \cos(\theta+90^\circ)=-\sin(\theta) \qquad \tan(\theta+90^\circ)=-\tan(\theta)$$

$$\sin(180^\circ-\theta)=\sin(\theta) \qquad \cos(180^\circ-\theta)=-\cos(\theta) \qquad \tan(180^\circ-\theta)=-\tan(\theta)$$

$$\sin(90^\circ-\theta)=\cos(\theta) \qquad \cos(90^\circ-\theta)=\sin(\theta) \qquad \tan(90^\circ-\theta)=\frac{1}{\tan(\theta)}=\cot(\theta)$$

$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b) \qquad \cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\sin(2a)=2\sin(a)\cos(a) \qquad \cos(2a)=\cos^2(a)-\sin^2(a)$$

$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right] \qquad \cos(a)\cos(b)=\frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]$$

$$\sin(a)\cos(b)=\frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

$$\sin^2\theta=\frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta=\frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a)=\frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

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4 Approximation Formulae

$$\frac{\Delta f}{\Delta x}\approx\frac{df}{dx} \qquad \frac{1}{1-x}\approx 1+x\ :\ (x<<1)$$

$$\sin\theta\approx\theta \qquad \tan\theta\approx\theta \qquad \cos\theta\approx 1-\frac{1}{2}\theta^2 \qquad \text{all for } \theta<<1$$

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5 Quadratic Formula

$$\text{If} \qquad 0=ax^2+bx+c\ , \qquad \text{then} \qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=-\frac{b}{2a}\pm\sqrt{\left(\frac{b}{2a}\right)^2-\frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\mathrm{m/s^2}$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}}t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 **Very Basic Newtonian Physics**

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 **Energy and Work**

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 **Momentum**

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total}\ f} = KE_{\text{total}\ i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin\gamma_L \qquad \tau_z = RF_{xy} \sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

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19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

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20 Gravity

$$\vec{F}_{1\text{ on }2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736 \times 10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5 \times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^{8} \approx 109 \times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

$$\begin{aligned}
\text{Pascal's principle} \quad & p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}} \\
\text{Archimedes principle} \quad & F_{\text{buoy}} = m_{\text{fluid dis}} g = V_{\text{fluid dis}} \rho_{\text{fluid}} g \\
\text{equation of continuity for ideal fluid} \quad & R_V = Av = \text{Constant} \\
\text{Bernoulli's equation} \quad & p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}
\end{aligned}$$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\text{T}}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_{\max}\sin[k(x \mp vt)] = y_{\max}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\max}^2$$

$$y = 2y_{\max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \ : \ n = 1, 3, 5, \ldots \qquad f_{\text{medium}} = \frac{f_0}{1 - v_0/v_{\text{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

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24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K} \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32°\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_{\text{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\text{RMS}}^2\right)$$

$$v_{\text{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^{\gamma} = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\mathrm{H}}} = \frac{Q_{\mathrm{H}} - Q_{\mathrm{C}}}{W} = 1 - \frac{Q_{\mathrm{C}}}{Q_{\mathrm{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\mathrm{H}}}{W} = \frac{Q_{\mathrm{H}}}{Q_{\mathrm{H}} - Q_{\mathrm{C}}} = \frac{1}{1 - Q_{\mathrm{C}}/Q_{\mathrm{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\mathrm{C}}}{W} = \frac{Q_{\mathrm{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\mathrm{C}}}{T_{\mathrm{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\mathrm{C}}/T_{\mathrm{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\mathrm{C}}/T_{\mathrm{H}}}{1 - T_{\mathrm{C}}/T_{\mathrm{H}}}$$