Intro Physics Semester I **Name:**

Homework 6: Newtonian Physics: More of the Same: One or two or no full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table Name:

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

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005 qmult 01200 1 4 5 easy deducto-memory: friction defined

1. "Let's play *Jeopardy*! For $100, the answer is: The macroscopic binding force between smooth surfaces that is parallel to the surfaces."

   What is ____________, Alex?

   a) the linear restoring force b) the spring force c) the tension force
   d) the normal force e) friction

   SUGGESTED ANSWER: (e)

   Wrong answers:
   b) Same thing as part (a).

   Redaction: Jeffery, 2008jan01

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005 qmult 01220 1 2 2 moderate memory: kinetic friction force

2. The magnitude of the kinetic friction force between a body and a surface equals a coefficient of friction times:

   a) the area of macroscopic contact between the body and the surface.
   b) the magnitude of the normal force acting on the body.
   c) the mass of the body.
   d) the density of the body.
   e) the density of the air surrounding the body.

   SUGGESTED ANSWER: (b)

   It is kinetic, not kinematic. It's a dynamic thing, not a kinematic thing. For kinetic friction,

   $$F_{\rm f} = \mu_{\rm ki} F_{\rm N} ,$$

   whereas for static friction,

   $$F_{\rm f} \leq \mu_{\rm st} F_N .$$

   The static friction equals the horizontal applied force that is trying to accelerate the object along surface until that applied force reaches the limit of $\mu_{\rm st}F_N$: once the body starts moving the kinetic friction law applies with coefficient $\mu_{\rm ki}$ and usually $\mu_{\rm ki} < \mu_{\rm st}$. Note these friction laws are actually approximations. They are often very good, but they arn't fundamental laws even in the restricted realm of classical physics.

   Wrong answers:
   a) Microscopically this is sort of true (WP, p. 103), but that is because the contact force between body and surface presses them together. The proportionality to the normal force is an experimental result that must be approximate at some level.
   c) No. If you push a body along the ceiling, the frictional force is completely independent of mass for example.
   d) Same as (c).
   e) Nope.

   Redaction: Jeffery, 2001jan01

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005 qmult 01230 1 2 5 moderate memory: friction coefficient sizes

3. Which is larger: the coefficient of static or kinetic friction?

   a) They are always equal.
   b) Neither. The larger depends on the materials involved and its about a 50-50 split on which is larger.
   c) The kinetic coefficient is always larger.
   d) The kinetic coefficient is usually larger.
   e) The static coefficient is almost always (always?) larger.

   SUGGESTED ANSWER: (e)

   The static coefficient is almost always larger. I hesitate to say always because perhaps there is some weird material interface where it isn't. One can sort of understand why static should be larger. In a sliding situation some microscopic bonds may not have a chance to form. Note the fact that the static coefficient is larger is why its recommended that in car slides on ice that you pump your breaks rather than locking them. This is so that the wheels keep rolling just fast enough to maintain a static friction resistance to motion. Actually, it is probably very difficult to do this trick optimally, and I've usually just lock my breaks on the few uncontrolled, but short, ice skids I've been in. And if you are skidding sideways into oncoming traffic (I wasn't at the wheel that time), you can't do much. But computerized systems can reduce skid distance by 30 % or more (reference ???).

Wrong answers:

Redaction: Jeffery, 2001jan01

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005 qmult 01240 1 1 3 easy memory: friction to heat

Extra keywords: physci related to KB-94-9 but not really that

4. The friction between sliding surfaces tends to change macroscopic kinetic energy into:

a) potential energy. b) rest mass energy. c) thermal or heat energy.
d) magnetic energy. e) nothing.

SUGGESTED ANSWER: (c)

Wrong answers:

e) This violates conservation of energy.

Redaction: Jeffery, 2001jan01

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005 qmult 01280 1 5 4 easy thinking: balanced forces on a coffee cup

5. You are pushing a coffee cup across a table (just an ordinary table, not an imaginary frictionless table) at a **CONSTANT** velocity. The magnitudes of the push force and frictional force are $|F_{\rm p}|$ and $|F_{\rm f}|$, respectively.

a) $|F_{\rm f}| > |F_{\rm p}|$ and this is why the cup does not accelerate.
b) The $|F_{\rm f}| < |F_{\rm p}|$, but nevertheless the frictional force prevents any acceleration.
c) There is no frictional force when you push the cup at a constant velocity. Thus, $|F_{\rm f}| = 0$ and clearly then $|F_{\rm p}| > |F_{\rm f}|$.
d) The $|F_{\rm f}|$ must **EQUAL** $|F_{\rm p}|$ in order for there to be no acceleration.
e) The $|F_{\rm f}|$ must be **TWICE** $|F_{\rm p}|$ in order for there to be no acceleration.

SUGGESTED ANSWER: (d)

Ah, *mes chers*, $\vec F_{\rm net} = m\vec a$ is always true and it's always true component by component. If there is no acceleration, there is no net force and the existing forces must be zero or cancel out.

Wrong answers:

a) I've lived in vain.

Redaction: Jeffery, 2001jan01

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005 qfull 01200 1 3 0 easy math: determining coefficient of static friction

6. Will will now determine the coefficient of static friction $\mu_{\rm st}$ from an empirical measurement using an ajustable incline. The adjustable angle of incline from the horizontal is θ.

a) Write down Newton's 2nd law for a block sitting at rest on the incline for two directions: perpendicular to the incline and parallel to it. The only forces are the gravity, the normal force, and friction.

b) The angle of adjustable incline is increased just to the slipping point for the block. Give the parallel 2nd law equation just before slipping occurs.

c) Solve for the $\mu_{\rm st}$.

SUGGESTED ANSWER:

a) Behold:

$$0 = F_{\rm N} - mg\cos\theta \ ,$$

$$0 = mg\sin\theta - F_{\text{st,f}} \ ,$$

where F_{N} is the normal force, $F_{\text{st,f}}$ is the static friction force, and the positive directions are down the incline for the x-coordinate and outward from incline for the y-coordinate.

b) Behold:

$$0 = mg\sin\theta - \mu_{\text{st}} mg\cos\theta \ ,$$

where we have used the rule that the maximum static friction force is given by $\mu_{\text{st}} F_{\text{N}}$ and the fact that in this case the normal force is given by $F_{\text{N}} = mg\cos\theta$.

c) Behold:

$$\mu_{\text{st}} = \tan\theta \ ,$$

where remarkably the gravitational field magnitude and mass have canceled out.

Redaction: Jeffery, 2008jan01

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006 qmult 00250 1 1 4 easy memory: change force contant change period for harmonic ocsillator

7. If you double the force constant of a simple harmonic oscillator, the period changes by a factor of:

a) $1/2$. b) $\sqrt{2}$. c) 2. d) $1/\sqrt{2}$. e) $2/3$.

SUGGESTED ANSWER: (d)

The period formula for a simple harmonic oscillator is

$$P = 2\pi\sqrt{\frac{m}{k}} \ .$$

If the initial force constant is k and it is double to get k_{new}, then

$$P_{\text{new}} = 2\pi\sqrt{\frac{m}{k_{\text{new}}}} = 2\pi\sqrt{\frac{m}{2k}} = \frac{P}{\sqrt{2}} \ .$$

Wrong answers:

a) A nonsense answer.

Fortran-95 Code

Redaction: Jeffery, 2008jan01

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006 qmult 00310 1 1 2 easy memory: 2nd law in polar coordinates

8. The formula

$$F_{r,\text{net}}\hat{r} + F_{\theta,\text{net}}\hat{\theta} = m\left[\left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}\right]$$

is:

a) Newton's 2nd law in spherical polar coordinates. b) Newton's 2nd law in polar coordinates.
c) the centripetal force formula. d) the centripetal acceleration formula.
e) the simple harmonic oscillator formula.

SUGGESTED ANSWER: (b)

Wrong answers:

a) Boy that's a complex formula.

Redaction: Jeffery, 2008jan01

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006 qmult 00400 1 1 4 easy memory: uniform circular motion defined

Extra keywords: physci

9. Uniform circular motion is motion in a/an:

a) circle at a constant **VELOCITY**. b) oval at a constant **VELOCITY**.
c) oval at a constant **SPEED**. d) circle at a constant **SPEED**.
e) circle at a nonconstant **SPEED**.

SUGGESTED ANSWER: (d) Uniform in the context of motion is often a synonym for constant.

Wrong answers:

a) I've lived in vain.

Redaction: Jeffery, 2001jan01

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006 qmult 00430 3 1 1 easy math: centripetal acceleration

10. The formula for the magnitude of centripetal acceleration is:

$$a_{\text{centripetal}} = \frac{v^2}{r} \ ,$$

where v is the speed of a uniform circular motion and r is the radius of the motion. Say $v = 10\,\text{m/s}$ and $r = 5\,\text{m}$, what is $a_{\text{centripetal}}$?

a) $20\,\text{m/s}^2$. b) $10\,\text{m/s}^2$. c) $5\,\text{m/s}^2$. d) $100\,\text{m/s}^2$. e) $15\,\text{m/s}^2$.

SUGGESTED ANSWER: (a)

Wrong answers:

Redaction: Jeffery, 2001jan01

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006 qmult 00450 1 4 3 easy deducto-memory: centripetal force defined

11. The centripetal force is:

a) a mysterious force that **APPEARS** whenever an object goes into uniform circular motion.
b) a mysterious force that trys to throw you **OFF** playground merry-go-rounds.
c) in fact $\vec{F}_{\text{net}}$ of $\vec{F}_{\text{net}} = m\vec{a}$ when this equation is specialized to the case of uniform circular motion. It is **NOT** a mysterious force that appears whenever you have uniform circular motion. Particular physical forces (e.g., gravity, tension force, and normal force) must act (sometimes in combination) to give a centripetal force which then causes uniform circular motion.
d) in fact $\vec{F}_{\text{net}}$ of $\vec{F}_{\text{net}} = m\vec{a}$ when this equation is specialized to the case of uniform circular motion. The force itself is **ALWAYS** a field force emanating from the center of motion that pulls on the circling object atom by atom.
e) a mysterious force that **DISAPPEARS** whenever an object goes into circular motion.

SUGGESTED ANSWER: (c)

At least it is easy with all the easy ways to eliminate wrong answers and the fact that it conforms to the longest-answer-is-right rule.

I the term centripetal force is reserved for circular motion, but that motion doesn't have to be uniform (i.e., constant speed) and it need only be a part of a circle. See Fr-108, 200, 557.

Wrong answers:

d) No it's not always a field force. It can be a tension force (which is a contact force) in the case of a sling for example. It can be friction or a normal force. Gravitational orbits are, of course, important cases where the force is indeed a field force.
e) All things are wrong.

Redaction: Jeffery, 2001jan01

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006 qmult 00472 3 5 1 tough thinking: lifting from a hump

Extra keywords: physci KB-61-39

12. There is a hump on the road with a cylindrical shape. The radius of the hump is 14.7 m. In an idealized picture, above about what horizontal speed must a car at the top of the hump lift from the hump?

a) 12 m/s. b) 14.7 m/s. c) 144 m/s. d) 10 m/s. e) 10.4 m/s.

SUGGESTED ANSWER: (a)

The car at the top of the hump is executing circular motion about the hump's center of curvature which, of course, is a point beneath the ground. But to execute circular motion there must be a centripetal force. The combination of gravity on the car and the normal force of the ground supplies the centripetal force on the car. But the ground normal force is the reaction force to the car normal force on the ground. As the car moves faster, there is less car normal force because more of the gravity force on the car is needed to keep the car moving in a circle and less is pushing the car into the road. The car normal force goes to zero when all of the gravity force is needed to maintain circular motion. But this means that the ground normal force goes to zero by the 3rd law. Now neither normal forces can be attractive. So when the car's speed exceeds the speed where the entire gravity force is needed to supply the centripetal force then there is not enough force to keep the car on the hump and it must lift.

The explanation is a bit complex, but the phenomenon of cars lifting as you drive to fast over humps is not uncommon.

The maximum car speed without lifting is given when the centripetal force is just the gravitational force alone:

$$mg = m\frac{v^2}{r}$$

which leads to

$$v = \sqrt{rg} = \sqrt{14.7 \times 9.8} \approx 144 = 12\,\mathrm{m/s}\ .$$

If the car speed exceeds about 12 m/s, the car tends to lift. Now 12 m/s is about 43 km/h or 27 mi/h which isn't all that fast. So this hump must have pretty small radius of curvature actually. But on the other hand, this is just the speed that gives the first bit of lift: the car isn't going to jump 20 tractors like Evil Knievel used to do.

Fortran-95 Code

```
        print*
        gg=9.8d0
        rr=14.7d0
        vv=sqrt(gg*rr)
        vv2=vv*(1.d0/1000.d0)*(3600.d0)
        vv3=vv2*(1.d0/1.609344d0)
        print*,'vv,vv2,vv3'
        print*,vv,vv2,vv3
!     12.0024997396376         43.2089990626953         26.8488272629688
```

Wrong answers:

c) Did you forgot to take the square root?

Redaction: Jeffery, 2001jan01

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006 qmult 00550 1 1 4 easy memory: banking angle formula

13. The banking angle formula is ____________. **HINT:** Use dimensional analysis. Ask yourself what formula has reasonable limiting behavior when input values go to extremes. Or just derive it using the centripetal force formula and the 2nd law.

a) $\theta = \tan^{-1}\left(\frac{v}{rg}\right)$ b) $\theta = \tan^{-1}\left(\frac{rg}{v^2}\right)$ c) $\theta = \tan^{-1}\left(\frac{v^2 r}{g}\right)$ d) $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$
e) $\theta = \tan^{-1}\left(\frac{g}{v^2 r}\right)$

SUGGESTED ANSWER: (d)

You should be able to rederive the result at the drop of a hat. One must have $F_{\rm nor}\sin\theta$ to supply the centripetal force and $F_{\rm nor}\cos\theta$ to cancel gravity. So one has

$$F_{\rm nor}\sin\theta = m\frac{v^2}{r} \qquad \text{and} \qquad F_{\rm nor}\cos\theta = mg\ .$$

Dividing the first by the second gives

$$\tan\theta = \frac{v^2}{rg}$$

and then

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)\ .$$

Wrong answers:

a) The argument of inverse tangent has the dimensions of T/L which can't be correct. The argument should be dimensionless.
b) If v goes to infinity, the banking angle goes to zero by this formula. But how likely is that.
c) The argument of inverse tangent has the dimensions of L^2 which can't be correct. The argument should be dimensionless.

Redaction: Jeffery, 2008jan01

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006 qmult 00770 2 5 2 moderate thinking: circular motion in non-inertial frame
Extra keywords: physci KB-59-21 , but I've corrected it

14. In what situations, if any, can a body move in a circular path at constant speed without a centripetal force?

a) None. b) In certain special non-inertial frames. c) In all non-inertial frames.
d) In all inertial frames. e) Always.

SUGGESTED ANSWER: (b)

Is there an example of such a special inertial frame? Sure. Consider a bug sitting anywhere on an old-fashioned record turntable. From the bug's perspective the whole outside world is going around him/her. But there is no centripetal force causing the whole outside world to do this.

As another example, consider the Earth-Sun system. From the Earth's point of view, the Earth is at rest and the Sun orbits the Earth. Geometrically this is perfectly true. But the Earth's frame is not inertial and the Sun's is—or at least is much more inertial than the Earth's. In the Sun's inertial frame the Earth orbits the Sun. The gravitational force of the Earth on the Sun causes this acceleration. The Earth's gravitational force on the Sun is equally strong in magnitude by the 3rd law, but because of the Sun's much greater mass, it's the Earth that orbits in the Sun's inertial frame. Actually, the Sun is somewhat accelerated around the center of mass of the solar system, somewhat accelerated around the center of mass of the Milky Way, somewhat accelerated around the center of mass of the Local Group (of galaxies), and somewhat accelerated in some other astronomical frames too.

Wrong answers:

a) This answer is wrong unless it is understood that the question is taken as referring to inertial frames, but I am not implying such an "understood." Clearly not since answer (b) is the right general answer.
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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006 qfull 00410 2 5 0 moderate thinking: Is-Hilda on vinyl record

15. Is-Hilda the ladybug is on a vinyl record spinning at 78 rpm. She starts at the center and six-leggedly walks radially outward. Her radial velocity (caused by her walking) is negligible compared to her tangential velocity (caused by the record motion). At 6 cm from the center, Is-Hilda suddenly slides off the record. The record counterfactually is a smooth surface—and isn't corrugated—and Is-Hilda is an ideal ladybug without sticky feet. What is the static friction coefficient between her and the record? What is the kinetic friction coefficient if you can determine it?

SUGGESTED ANSWER:

In order to be carried in uniform circular motion by the record there has to be a centripetal force. In this case, it is supplied by static friction as Is-Hilda walks. At the point of sliding this static friction must have just reached it's maximum value. Thus

$$F_{\text{st max}} = \mu_{\text{st}} F_{\text{N}} = \mu_{\text{st}} mg = F_{\text{cen}} = m\frac{v^2}{r} = mr\omega^2 \ .$$

Solving for the static friction coefficient gives

$$\mu_{\text{st}} = \frac{r\omega^2}{g} \approx \frac{0.06 \times 8^2}{10} \approx 0.4$$

to 1-digit accuracy. To better accuracy $\mu_{\rm st} = 0.41$. We really don't have the information to deduce the kinetic friction coefficient. But we can suspect that it must be less than the static one (as is the usual case) or else Is-Hilda would probably start to slide and then sort of stop.

Where does the 8 come from in the above calculation? Behold:

$$\omega = 2\pi f = 2\pi \times 78\,{\rm rev/min} \times \left(\frac{1\,{\rm m}}{60\,{\rm s}}\right) \approx 8\,{\rm rad/s}\ .$$

More exactly, one gets $\omega = 8.16\,{\rm rad/s}$.

Fortran Code

```
        print*
        pi_con=acos(-1.d0)
        frpm=78.d0
        g=9.8d0
        r=0.06d0
        omega=2.d0*pi_con*frpm/60.d0
        xmu=r*omega**2/g
        print*,'omega,xmu'
        print*,omega,xmu
!       8.16814089933346       0.408480769906310
```

Redaction: Jeffery, 2001jan01

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006 qfull 00420 1 3 0 easy math: conical pendulum

16. There is a conical pendulum of length ℓ with bob of mass m. The bob is executing uniform circular motion with velocity v and radius r. The pendulum has an ideal mass rope with tension T. The angle of the rope from the horizontal is θ.

a) Write down Newon's 2nd law for the horizontal directions and vertical directions making use of the centripetal acceleration formula.

b) For a person just swinging a conical pendulum by hand, it is probably easiest to set θ (at least roughly), ℓ and m. Find formulae for the variables r, T, and v as functions of **ONLY** θ, ℓ, m and g. Write the T formulae with θ only appearing in the sine function.

c) Give the formula for the period P of the motion as a function of **ONLY** θ and ℓ.

SUGGESTED ANSWER:

a) Behold:

$$m\frac{v^2}{r} = T\cos\theta\ ,$$
$$0 = T\sin\theta - mg\ .$$

b) For r behold:

$$r = \ell\cos\theta\ .$$

For T behold:

$$T = \frac{mg}{\sin\theta}\ .$$

For v behold:

$$\begin{aligned} mg &= T\sin\theta \\ m\frac{v^2}{r} &= T\cos\theta\ , \\ \frac{rg}{v^2} &= \tan\theta\ , \\ \frac{g\ell\cos\theta}{v^2} &= \tan\theta\ , \\ v &= \sqrt{g\ell\left(\frac{1-\sin^2\theta}{\sin\theta}\right)}\ . \end{aligned}$$

c) Behold:

$$P = \frac{2\pi r}{v} = \frac{2\pi\ell\cos\theta}{v} = 2\pi\sqrt{\frac{\ell}{g}\sin\theta}$$

or concisely

$$P = 2\pi\sqrt{\frac{\ell}{g}\sin\theta}\ .$$

Remarkably in the limit that $\theta \to 0$, the period becomes

$$P = 2\pi\sqrt{\frac{\ell}{g}}$$

which is also the formula of the simple pendulum which is executing a 2-dimensional swinging oscillation. I guess all this means is that the two motions are degenerate in this limit: a circular motion of zero radius is the same as swinging oscillation of zero amplitude.

Redaction: Jeffery, 2008jan01

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006 qfull 00510 1 3 0 easy math: level road corner centripetal force

17. Consider an ordinary road corner that is level ground. Such corners are usually not banked very much, in fact.

a) What force supplies the centripetal force that allows a vehichle to make the the corner?

b) For a corner turn with radius of curvature r and a vehicle of mass m, derive the formula for the maximum turn speed v before slipping occurs.

c) Say $r = 10\,\mathrm{m}$ (which seems reasonable for ordinary corners) and $\mu_{\mathrm{st}} = 1.0$ (as for rubber on concrete in dry conditions approximately), what is the maximum turn speed before slipping? Now say $\mu_{\mathrm{st}} = 0.30$ (as for rubber on concrete in dry conditions approximately), what is the maximum turn speed before slipping? Convert the answers to miles per hour.

SUGGESTED ANSWER:

a) Static friction. The road pushes on the car tires in the horizontal direction using friction. Since the car should not be skidding, it is static friction. Of course, if the car is skidding, the friction is kinetic friction. The road friction is a reaction force to the car putting a friction force on the road.

b) For maximum turn speed, the static friction force has reached its upper limit. Thus,

$$m\frac{v^2}{r} = \mu_{\mathrm{st}} mg\ ,$$

and so

$$v = \sqrt{\mu_{\mathrm{st}} gr}\ .$$

c) Well

$$v = \sqrt{\mu_{\mathrm{st}} gr} = \sqrt{\mu_{\mathrm{st}}\left(\frac{r}{10\,\mathrm{m}}\right)} \times 9.90\,\mathrm{m/s} = \sqrt{\mu_{\mathrm{st}}\left(\frac{r}{10\,\mathrm{m}}\right)} \times 22.1\,\mathrm{mi/h}\ .$$

Thus, the maximum speed for $\mu_{\mathrm{st}} = 1.0$ is $9.90\,\mathrm{m/s} = 22.1\,\mathrm{mi/h}$ and for $\mu_{\mathrm{st}} = 0.30$, it is $5.42\,\mathrm{m/s} = 12.2\,\mathrm{mi/h}$. Clearly, under wet conditions one should take corners more slowly than under dry.

Fortran-95 Code

```
print*
gg=9.8d0
r=10.d0
con=(1.d0/1609.344d0)*(3600.d0)
xmu=1.d0
vmax=sqrt(xmu*gg*r)
```

```
        vmaxa=vmax*con
        vmax1=sqrt(0.30d0*gg*r)
        vmax1a=vmax1*con
        print*,'vmax,vmaxa,vmax1,vmax1a'
        print*,vmax,vmaxa,vmax1,vmax1a
  ! 9.8994949366116654  22.144539496715428  5.4221766846903838
12.129063807915138
```

Redaction: Jeffery, 2008jan01

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006 qfull 00550 2 3 0 moderate math: plane making a banked turn

18. An airplane of mass m is making a turn through a circular bend with a speed of v. It moves horizontally only: i.e., it's not moving or accelerating downward or upward. In order to make the turn the plane banks at angle θ from the horizontal (i.e., its wings are tipped at θ to the horizontal with the high side away from the center of the turn). Assume all the force the air can exert on the plane (the aerodynamic lift) acts perpendicularly to the wings. Let F_{lift} be the aerodynamic lift magnitude.

a) Draw a free body diagam for the airplane in the plane perpendicular to the airplane's instantaneous direction of motion.

b) Apply Newton's 2nd law to airplane in the plane perpendicular to the airplane's instantaneous direction of motion. Remember the course mantra:

"Newton's 2nd law is always true and it's always true component by component."

c) Solve for the r in terms of the other variables: i.e., in terms of m, θ, v, g, and F_{lift}.

d) Given $v = 600\,\text{km/h}$ and $\theta = 30°$, what is r?

SUGGESTED ANSWER:

a) You will have to imagine the free body diagram.

b) It makes sense to put the x axis in the horizontal direction and the y axis in the vertical direction. Since the airplane is moving in a circle, the ma of $F = ma$ is the centripetal force. Thus, in the horizontal direction we have

$$\frac{mv^2}{r} = F_{\text{lift}} \sin\theta \ .$$

In the vertical direction, we have

$$0 = F_{\text{lift}} \cos\theta - mg \ .$$

c) From the part (b) answer, we have

$$\frac{mv^2}{r} = F_{\text{lift}} \sin\theta \qquad \text{and} \qquad mg = F_{\text{lift}} \cos\theta \ .$$

Dividing the first by the second gives

$$\frac{v^2}{rg} = \tan\theta$$

which is our old friend the banking angle formula. That this formula turns up again should be no surprise. We have essentially the same setup as for a car turning on a round with the road banking adjusted so that the centripetal force must entirely supplied by the component of a force that is perpendicular to the vechicle.

Solving for r gives

$$r = \left(\frac{v^2}{g}\right)\left(\frac{1}{\tan\theta}\right) \ .$$

d) First, we did have to convert 600 km/h to its meters per second equivalent to do the calculation. Behold:

$$600\,\text{km/h} = 600\,\text{km/h} \times \left(\frac{1000\,\text{m}}{1\,\text{km}}\right) \times \left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 167\,\text{m/s} \ .$$

Now

$$r = \left(\frac{v^2}{g}\right)\left(\frac{1}{\tan\theta}\right) \approx 5\,\text{km}$$

to about 1-digit accuracy. More exactly

$$r = 4.9\,\text{km} \ .$$

Fortran Code

```
      print*
      pi_con=acos(-1.d0)
      raddeg2=180.d0/pi_con
      vv1=600.d0/3.6d0
      vv2=480.d0/3.6d0
      thet1=30.d0
      thet2=40.d0
      gg=9.8d0
      rr1=(vv1**2/gg)/tan(thet1/raddeg2)
      rr2=(vv2**2/gg)/tan(thet2/raddeg2)
      print*,'vv1,vv2'
      print*,vv1,vv2
!  166.666666666667        133.333333333333
      print*,'rr1,rr2'
      print*,rr1,rr2
!     4909.44141  2161.91162
```

Redaction: Jeffery, 2001jan01

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006 qfull 00700 1 3 0 easy math: general drag force in falling from rest case

19. An object falling from rest is subject to a drag force of magnitude $f(v)$, where v is the object's speed. The function $f(v)$ is monotonically increasing with v and $f(0) = 0$, but is otherwise general. The inverse function to $f(v)$ is $f^{-1}(x)$.

   a) Apply Newton's 2nd law to the object taking the downward direction as positive. **HINT:** You just write down the equation.

   b) What is terminal velocity and why should the falling object reach it? What is the formula for the terminal velocity v_{ter}? Make use of $f^{-1}(x)$.

   c) The evolution to terminal velocity can be crudely divided into two phases: a linear growth phase (when the velocity is growing approximately linearly) and an asymptotic phase (when the velocity is asymptotically approaching terminal velocity). The characteristic time t_{ch} border between the two phases is obtained by setting $f(v) = 0$ in the equation of motion and solving for the time when $v = v_{\text{ter}}$. Beyond this time, the pure linear growth must be over. Derive the formula for t_{ch}. Then derive the formula for the characteristic length ℓ_{ch} which is the distance fallen in time t_{ch} assuming $f(v) = 0$.

   d) The formula

   $$v = v_{\text{ter}}(1 - e^{-t/t_{\text{ch}}})$$

   is a crude approximate solution for velocity in general. If we have linear drag $f(v) = bv$ (where b) is a constant, then the solution becomes exact. There are cases where linear drag holds: for very low speeds and no turbulence. Sketch a plot of the approximate solution. Verify by substitution into the equation of motion (i.e., our Newton's 2nd law application) that the velocity formula is exact for $f(v) = bv$.

SUGGESTED ANSWER:

a) Behold:

$$mg - f(v) = ma \ .$$

b) Terminal velocity occurs when the gravitational force and the drag force cancel. The object is then moving a constant velocity which is the terminal velocity. An object should reach terminal velocity. Initially $f(v)$ is zero and the object accelerates. As v increases, $mg - f(v)$ decreases, but stays positive and so the acceleration is positive and velocity continues to increase. But when $mg - f(v)$ reaches zero, the acceleration goes to zero, and the object's velocity becomes constant and stays constant. This constant velocity is the terminal velocity.

It takes a mathematical analysis to show this, but for usual drag forces terminal velocity is formally only reached at time equals infinity. The velocity asymptotically approaches terminal velocity. In reality, the fluid medium is subject to velocity fluctuations and once the difference between object velocity and terminal velocity becomes smaller than these, the object has effectively reached terminal velocity. This happens usually at much less than time equals infinity.

The terminal velocity formula is

$$v_{\rm ter} = f^{-1}(mg) \ .$$

c) If

$$mg = ma \ ,$$

then

$$v = gt \ ,$$

and so

$$t_{\rm ch} = \frac{v_{\rm ter}}{g} \ .$$

The characteristic length is

$$\ell_{\rm ch} = \frac{1}{2} g t_{\rm ter}^2 = \frac{v_{\rm ter}^2}{2g} \ .$$

d) You will have to imagine the sketch. Now for the verification. In this case,

$$v_{\rm ter} = \frac{mg}{b} \qquad \text{and} \qquad t_{\rm ch} = \frac{m}{b} \ .$$

Thus, the equation of motion can be written

$$v_{\rm ter} - v = a t_{\rm ch}$$

or

$$v = v_{\rm ter} - a t_{\rm ch} \ .$$

Now from the velocity solution, we find that

$$a = \frac{v_{\rm ter}}{t_{\rm ch}} e^{-t/t_{\rm ch}} \ .$$

Substituting this formula for a into the equation of motion gives

$$v = v_{\rm ter}(1 - e^{-t/t_{\rm ch}})$$

which is consistent with our assumed solution. This verifies that the assumed solution is exact for the case of linear drag.

Redaction: Jeffery, 2008jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67428(67) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2006, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1 - x} \approx 1 + x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

$$\text{If} \qquad 0 = ax^2 + bx + c\,, \qquad \text{then} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series
$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\mathrm{m/s^2}$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\, dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$