ii) Gauss' Law and Shell Theorem Analogs for the linear Force

For inverse square law forms see p. 3005 — 3014e

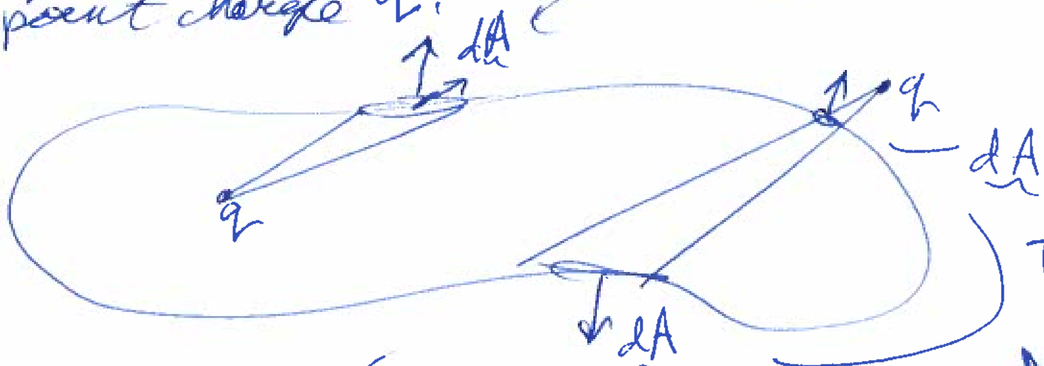
$$\underset{\sim}{f} = \underbrace{q}_{\text{can be +ve or -ve}} r\hat{r}$$

We assume $r \to \infty$ and so $f \to \infty$ as $r \to \infty$ which is a hard to understand ~~what~~ ~~they~~ that means in all physical senses

a force field — a force per unit charge for point charge q.

Sending and Receiving charge may not be the same thing necessarily, but the 3rd law implies they are

surface area vector



These do Not cancel ~~in gen~~ in general since they require the inverse square law

Divergence Theorem (see Wik and p. 3008) or Gauss Theorem

$$\underbrace{\oint \underset{\sim}{f} \cdot d\underset{\sim}{A}}_{\text{over whole surface closed}} = \int \nabla \cdot \underset{\sim}{f}\, dV$$

$$\nabla \cdot \underset{\sim}{f} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 f\right)$$

for spherical symmetry (Wik: Divergence) and spherical coordinates

$$\nabla \cdot f = \nabla \cdot (q r \hat{r})$$
$$= q \frac{1}{r^2}\frac{\partial}{\partial r}(r^3) = 3q$$

$\therefore \oint \underset{\sim}{f} \cdot dA = 3qV$ whether q is inside or outside and wherever q is in the universe!!

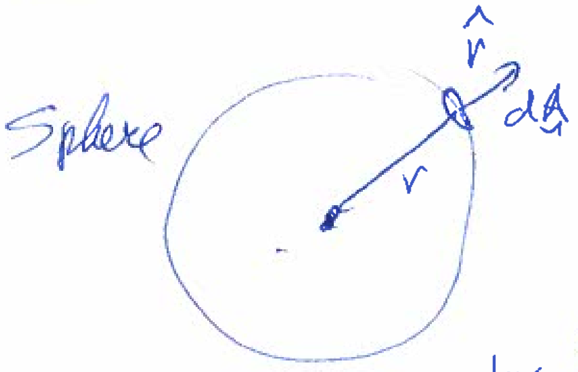
$$\therefore \oint \underbrace{\underset{\sim}{f}}_{\text{field}} \cdot dA = 3QV$$

for any distribution of charges throughout universe that sums to Q.

30ty8

As with Gauss' law the analog can be used to solve the field directly for the 3 cases of high symmetry a) spherical, b) clyindrical, c) planar.

Shell Theorem analog



and Q is the sum of all charge spread symmetrically thru all space.

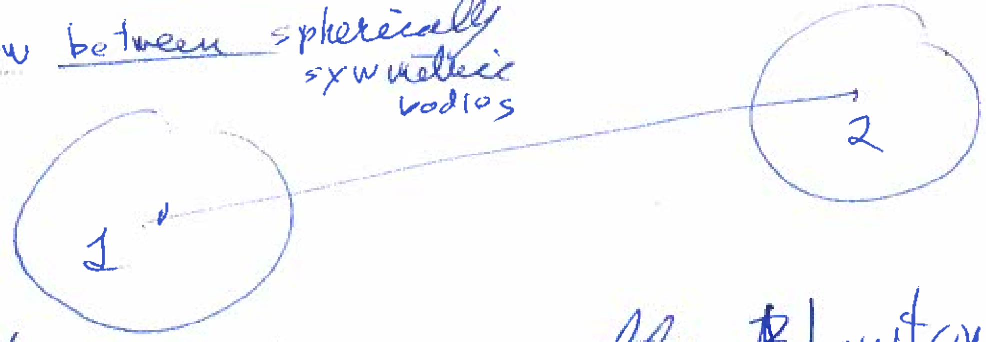
$$\oint \underset{\sim}{f} \cdot d\underset{\sim}{A} = 3VQ$$

by symmetry

$$f \, 4\pi r^2 = 3\underbrace{Q\left(\frac{4\pi}{3} r\right)}_{\text{charge inside and outside of sphere}}$$

$$\underset{\sim}{f} = Q r \hat{r} \quad \text{GED}$$

Now between spherically symmetric bodies



We have to assume the Newton's 3rd law holds for the linear force.

Probably $\underset{\sim}{F}_{12} = q_1 q_2 r_{12} \hat{r}_{12}$ is needed for point charges.

so as
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to bodies
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ach
other
ie gravity
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though not
thers) are
int
like.

$\underset{\sim}{F}_{12} = F_{1 \text{point}, 2}$ by shell theorem

$= -F_{2, 1 \text{point}}$ by 3rd law

$= -F_{2 \text{point}, 1 \text{point}}$ by shell theorem

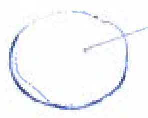
$= F_{1 \text{point}, 2 \text{point}}$ by 3rd law.

Given $\underset{\sim}{F}_{\substack{\text{external net} \\ \text{on } 2}} = m_2 a_{\text{center of mass } 2}$, the force of 1 on 2 is ~~exactly as for point masses as long as~~ no non ~~point like effects~~

But what if the charge distribution were spread evenly thru infinite space

$$Q \Rightarrow \infty \;?$$

Assume result



needs to infini

- and any

We're flummoxed

But there's the old renormalization trick. If a quantity goes to ∞, just make it a free parameter and set it to what you want.

point c
be an
origin

Justification = fudge

~~or~~ Or your theory is an emergent ~~theory~~ - a correct limit of a more general ~~theory~~ you don't know, but if you did you could take the limit.

We don't explicitly need the 3rd law for this way of getting the cosmological constant force.

What is the cosmological constant force?

If you write it in the classical limit as a force it is $F_\Lambda = \frac{\Lambda}{3} m r \hat{r}$ where m is mass of test particle, Λ is Einstein's cosmological consta

$Q \rightarrow$ our Q

$[\Lambda] = \frac{E}{ML^2} = T^{-2}$

$V = -\frac{1}{2}\frac{\Lambda}{3} m r^2$

3060

But perhaps this linear force perspective is NOT a fundamental one and it is unfruitful for understanding at a deeper level.^{2} a) For one thing, how does Birkhoff's theorem apply to such a linear force? Not clear to me. So another POV will be used. → We Just assume we can use the linear force. Maybe renormalization trick or on p. 30?.

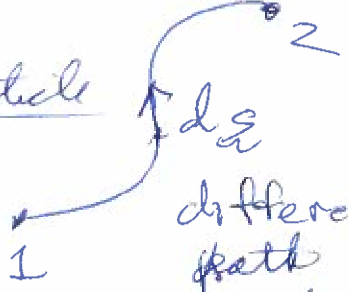
b) For another thing — $F_\Lambda \propto m$ like gravity — so maybe it really is gravity in a way we'll see!

Work-Energy Theorem & Conservation of Mech. Energy

- Classical result.
- but we can use it since Birkhoff's theorem permits & apparently GR in the weak field low velocity limit (

Start with where one always starts in classical mechanics

$$\underset{\sim}{F}_{net} = m\underset{\sim}{a} \quad \text{for a particle}$$



differential path vecto

$$dW = \underset{\sim}{F}_{net} \cdot d\underset{\sim}{s} = m\underset{\sim}{a} \cdot d\underset{\sim}{s}$$

$$= m \frac{d\underset{\sim}{v}}{dt} \cdot \underset{\sim}{v}\, dt$$

$$= \frac{1}{2} m \frac{dv^2}{dt} dt$$

$$dW = \frac{1}{2} m\, dv^2$$

$$W = \Delta KE$$ Work-Kinetic energy theorem.

Sort of time indefinite since $\cancel{dt = \frac{ds}{v}}$ $t = \int \frac{ds}{v(s)}$ $s(t)$ for init condit v_1

$$W = W_{con} + W_{non}$$

A conservative force

$$\underset{\sim}{F} = -\nabla \underset{\sim}{U}$$

nonconservative force work

Can calcula $W = \int \underset{\sim}{F} \cdot d\underset{\sim}{s}$ and know ΔKE but nee to sol $F = $ to gi $s(t)$ and Δt_{12}

negative sign needed to impose energy conservation

a potential energy — an energy of position in some sense as discussed p. 304 ff

$$\int \underset{\sim}{F} \cdot ds$$

$$\underset{\sim}{F} \cdot d\underset{\sim}{s} = -\nabla \underset{\sim}{U} \cdot ds = -\Delta U$$ along path

$$W_{con} = -\Delta U$$

Can be much

$$W = -\Delta U + W_{non}$$

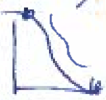
mechanical energy

$$\therefore \Delta E = \Delta KE + \Delta U = W_{non}$$ Work-Energy theorem

If $W_{non} = 0$, $\Delta KE + \Delta U = 0$, $E_{mech} = KE + U$

or $\Delta KE = -\Delta U$

- Recall Intro physics Mechanics energy conservation given speed ~~at~~ rolle unit mod $s(t)$



→ Hard to solve for time from F = ma -

3062a

3.1 Friedmann Equation

We've had a long warm up with various bits of physics and considerations

[Jump to p. 3062b]

Consider an ~~infinite~~ Unbounded, homogeneous isotropic universe

We only know of curvature of 3-d space from GR — but that means the space can be finite — like the 3-d surface of a sphere in 4-d Euclidean space

inite
ut
oundless
ike
urface
phere

Called a 3-sphere
(Wiki: n-sphere or hypersphere)

An ordinary sphere is a 2-sphere.

We consider a ~~2 sphere su~~ small enough region that 3-d space is flat or Euclidean.

Inertial Frames (Inertial reference frames = IRFs)

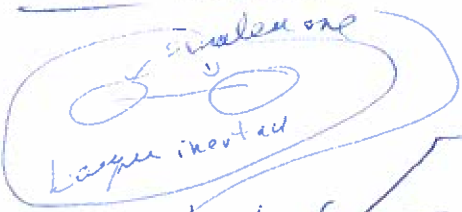
306

are Frames with respect to which all physical laws are defined — except in our modern view, GR which tells us what inertial frames are.

— You could quibble about this → e.g., what of the 2nd law of Thermodynamics → but after quibbling.. Quibbling may be definitional or matter of perspective/convenience

GR tells us they are free-fall frames ⟷ under mass-dependent forces
- gravity
- cosmological constant force → important only on cosmic scale
- inertial forces in non-inertial frames

But all ~~in~~ such de Not rotat with respect observ univer



Newtonian laws of motion at least with force gravity holds between inertial frames in NR limit

let's just concentrate on this case not to go down the rabbit hole of generality

Hold in classical limit absolute rotation

- No Absolute space
- But absolute rotation

classical limit

invented the nonce term COMFFI frames

Center-of-mass-free fall inertial frames Just in NR limit

(which planet people know more about than me

System of particles i

— could be anything

~~of~~ a grav. bound eg. planetary system

— a pressure supported system (eg. planet, star)

$$① \quad \sum_{\substack{j \\ j\neq i}} \underset{\sim}{F}_{ji} + \underset{\sim}{F}_{ei} + m_i \underset{\sim}{g}_i = m_i \underset{\sim}{a}_i$$

force of particle j on particle i

grav and all other internal force

all external forces except gravity

Just external gravity

mass of particle i

acceleration relative to some externally defined inertial frame ↳ we will make it disappear!!

$\boxed{3064}$ sum over i

② $$\underbrace{\sum_{\substack{ij \\ i\neq j}} \underset{\sim}{F}_{ij}}_{0} + \underbrace{\sum_i \underset{\sim}{F}_{ei}}_{\underset{\sim}{F}_e} + \underbrace{\sum_i m_i \underset{\sim}{g}_i}_{m\left(\frac{\sum_i m_i \underset{\sim}{g}_i}{m}\right)} = \underbrace{\sum_i m_i \underset{\sim}{a}_i}_{m a_{cm}}$$

Just external gravi

0 by pairwise cancellation of 3rd law (recall in classical limit)

$\equiv \underset{\sim}{g}_{ave}$

total mass

$$\underset{\sim}{F}_e + m \underset{\sim}{g}_{ave} = m \underset{\sim}{a}_c$$

if $\underset{\sim}{F}_e = 0$, $\underset{\sim}{a}_{cm} = \underset{\sim}{g}_{ave}$

Now $\frac{①}{m_i} - \frac{②}{m}$

Equation ① minus Eq ②

③ $$\underbrace{\sum_j \frac{\underset{\sim}{F}_{ji}}{m_i}} + \underbrace{\frac{\underset{\sim}{F}_{ei}}{m_i} - \frac{\underset{\sim}{F}_e}{m}} + \underbrace{\underset{\sim}{g}_i - \underset{\sim}{g}_{ave}}_{\text{tidal force}} = \underset{\sim}{a}_i - \underset{\sim}{a}_{cm} = \underset{\sim}{a}_i'$$

only ext.

zero if no external grav forces

zero if external gravity uniform

relative to CM

$$\underbrace{\underset{\sim}{g}_{ti}}_{\text{tidal force}} = \underset{\sim}{g}_i - \underset{\sim}{g}_{ave} = (\underset{\sim}{g}_i - \underbrace{\underset{\sim}{g}}_{\text{gravitational field at CM}}) - \underbrace{(\underset{\sim}{g}_{ave} - \underset{\sim}{g})}_{= \frac{\sum_i m_i (\underset{\sim}{g}_{ave} - \underset{\sim}{g})}{m}}$$

$$g_{ti} \simeq -\frac{GM}{r_{cm}^2}\left(\frac{1}{\left(1+\frac{\Delta r}{r_{cm}}\right)^2} - 1\right)$$

$$\simeq -\frac{GM}{r_{cm}^2}\left(2\frac{\Delta r_i}{r_{cm}}\right) \doteq -\frac{GM}{r_{cm}^3}\Delta r_i \sim \frac{1}{r_{cm}^3}$$

Note worst case for ~~grav~~ tidal force

$$\int_{r_1}^{r_2} \frac{GM}{r_{cm}^3} 4\pi r_{cm}^2 \, dr_{cm} \doteq \ln\left(\frac{r_2}{r_1}\right)$$

logarithmic divergence

it actually averages to zero

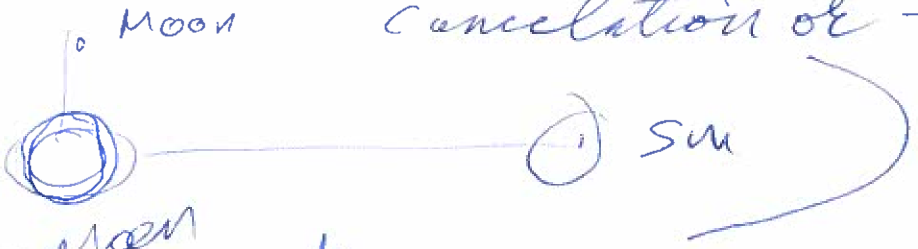
This can be used to calculate tidal bulges and the like (make a homework question someday)

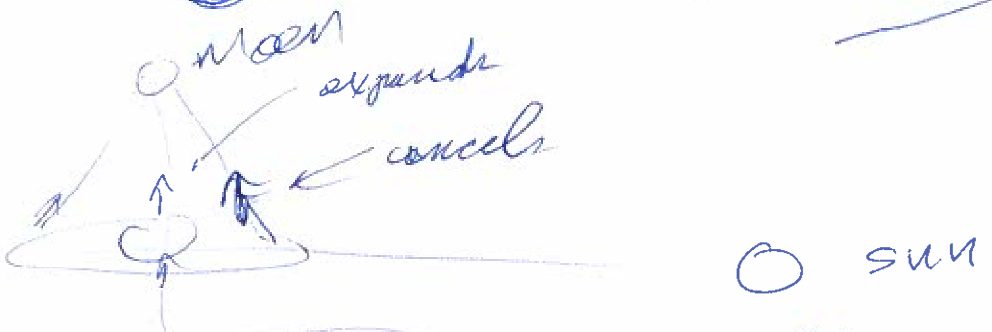
often very small for spherically symmetric bodies and can be set to zero to 1st order for stars planets

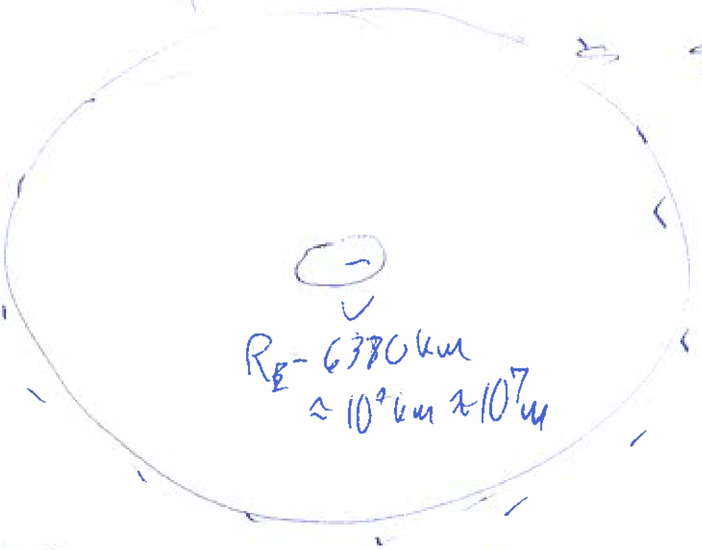
But does tidal force average to zero? 3065

spring tide, Not done

cancellation or → average expans



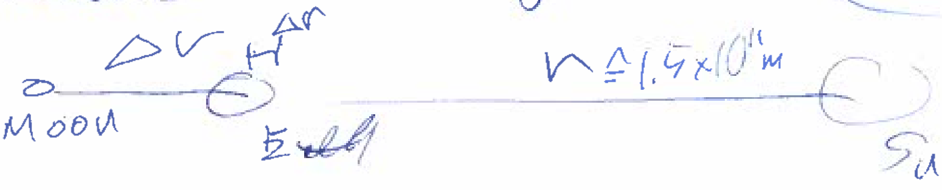




→ shell thus Birkhoff tr

handware to cancelation of gravity / ergo of ~~[illegible]~~ increasing tidal for as one goes to infinit

When is tidal force important



$$g_t \sim M_\odot \frac{\Delta r}{r^3} = \frac{2\times10^{30}\ \text{kg}}{(1.5\times10^{11}\text{m})^3}\Delta r_E$$

a scaled tidal force

$\Delta r_E \simeq$

$\sim .5\times10^{-3}$ or $\sim .5\times(1$

3066

for $\Delta r_E = 6370 \text{ km} \simeq 10^7 \text{ m}$

g_{ti} gives tides ~ 1 m in open ocean

counting Sun & Moon
Sun alone ~ $\frac{1}{3}$ M

Could calculate this to order of
(Make a problem some day)

$\Delta v_{\text{Earth-Moon}} \simeq 60 \times \Delta v_{\text{~~Earth~~ orbit}}$

— bigger → probably a significant perturbation, but ~~there~~ are others
(oblateness of Earth & Moon, other planets)

$C_\odot \simeq 1.5 \times 10^5$

What of ~~Earth~~ star in Milky ~~way~~ and cosmologically?

$$\Delta r \int_{r_1}^{r_2} \frac{\rho}{r^3} 4\pi r^2 dr$$

{ for upper limit assume – no cancellation

$C = \Delta r \; \rho \; 4\pi \ln(r_2/r_1)$

~~$\Delta r C = 30 \log(r_2/r_1)$~~

3 log x ~ 30

~~Δr_{Earth} 7.4 $\times 10^3 \cdot \rho \cdot 30$~~

~~$\Delta r_{\text{Earth}} \cdot 0{-}4 \log(r_2/r_1)$~~

$9 \times 10^{-20} \frac{\text{kg}}{\text{m}^3}$

Milky way
— in solar nbhd

By which location alerting kills it

(will – stellar density and Dark Matter included)

Even for $r_1 = 1$ ly
$r_2 = 10^9$ ly

By which

$C = 4\pi$ ... 10^{-8}

10^{-5}

Cluster stars

$$C = \Delta V \cdot \rho \cdot \underbrace{4\pi \cdot 2.3}_{\sim 30} \log(V_2/V_1)$$

[306

$\ln x \cong 2.3 \log$

$\rho_{crit} = 10^{-26}\ kg/m^3$ real cosmic dens of this order

$30 \times 4\times10^{-20} = 10^{-18}$ for Milky Way Density

$\rho_{MW} = 4\times10^{-21}\ kg/m^3 * 10 = 4\times1$

(stellar density (Wik))

Dark matter factor

$C^{\text{upper limit}} = \Delta V \cdot 10^{-18} \cdot \log(V_2/V_1)$

say $\frac{1\ Gly}{1\ ly} \sim 10^2$

1 Gly is worse $R_{ob} \approx 46$ G in proper length

$= \Delta V \cdot 10^{-17}$

so even with No cancellation due to spherical averaging (shell/Birkhoff theorem)

ΛCD model

Recall Sun tidal force at Earth

$C \sim .5\times10^{-3}\Delta V$

1 Gly is pushing Newtonian gravity too far since grav effect propagate at speed of light

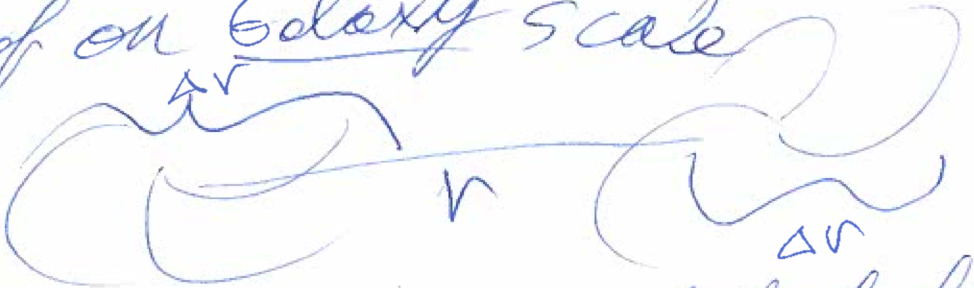
So tidal force of rest of universe on whole solar due to Shell/Birkhoff theorem cancellation $<< \Delta V \cdot 10^{-17}$

$\Delta V \sim 100 \cdot 1AU$
$\sim 100 \cdot 1.5\times10^{11}$
$\sim 1.5\times10^{13}$ [m

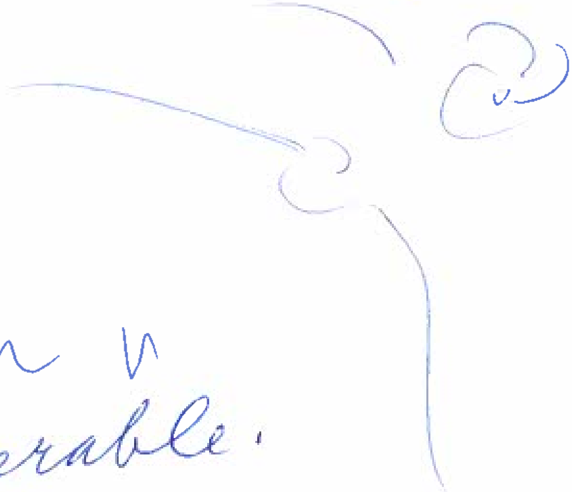
is minute even in vast overestimate

066 b 1

What of on Galaxy scale



Actually we know tidal forces are significant in these cases since interacting galaxies show tidal tails.



(e.g., Mice Galaxies)

$\Delta r \sim r$
comparable.

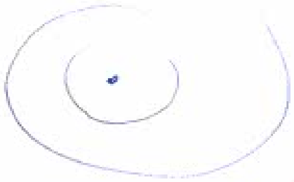
So no calculation here.

3067

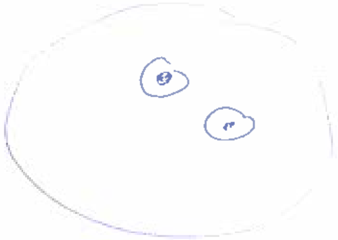
So tidal force of sun & planets in planetary system important but not ~~that~~ of rest of universe.

Same true of all isolated planetary system

ii) Planetary System isolated → COM FI frame ~~each~~ both of them



single star



Binary

free-fall in average $\underset{\sim}{g}$ of universe, but no tidal force from outside is significant

~~And~~ no other F_{e_i} significant
→ pressure
— magnetic field

So equation ③ on p. 3064 reduces to
with no g_{t_i} and no F_{e_i}

$$\sum_j \frac{\underset{\sim}{F}_{ij}}{m_i} = \underset{\sim}{a}_i'$$

No need to consider $\underset{\sim}{a}_i$ at all, for internal motions

gravity & internal pressure support forces

Earth in free fall – so earth inertial frame defined by CM → but we have to consider rotation at surface

3068)

This was known to celestial mechanics people from Newton on

or they wouldn't have got the right answer

But up until GR in 1915

People thought of Newton's absolute space

S One unique inertial frame → those not accelerated with respect to it.

all we
currec
~~have~~ to
ith physicists
e free-fall
rames
not rotating
th respect
ervable universe) are really true inertial frames
r
laif

For COMFFI frames → they thought of as noninertial

↳ they said $a_{cm} = g_{ave}$ frame

movies
f~~rames~~
CMs of
oid galaxies
d galaxy
asters
proximately

minus acceleration = $-a_{cm}$, $0 = -a_{cm} + g_{ave}$

as an inertial force to cancel gravity field.

so converted to inertial frame

Both perspectives ~~At~~ give some classical answer

I ~~often~~ say
~~Newtonian~~
physics classical
limit Relativistic
physics - true

But GR perspective is the true one (not just a classical limit since there is no absolute space

but no absolute space is always a wrong hypothesis

but absolute rotation of observable universe

ree
fall
frames
unrotating
with respect
to observable
universe with
no inertial forces
invoked are the basic inertial frames of the

→ and the basic are the comoving frames participating in expansion of universe

b) Specialize to 2-objects in ~~that~~ case (a)

$$\frac{F_{12}}{m_2} = a_2' , \quad \frac{F_{21}}{m_1} = a_1'$$

Relative acceleration

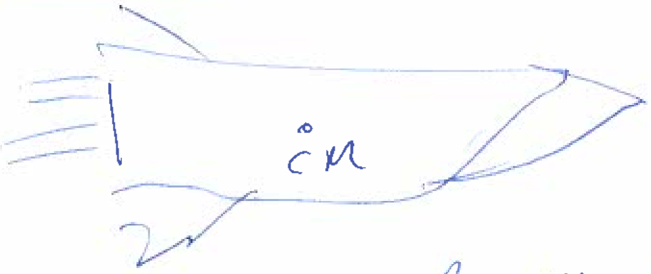
$$\Delta a = a_2' - a_1' = \frac{F_{12}}{m_2} - \frac{F_{21}}{m_1}$$

$$= F_{12}\left(\frac{1}{m_2} + \frac{1}{m_1}\right)$$

$$= F_{12} \underbrace{\frac{1}{M_{reduc}}}_{\text{old friend reduced mass}}$$

c) Rocket in space

Here we want



$$\underset{\sim}{F}_e + \underbrace{m \underset{\sim}{g}_{ave}}_{\text{field } \sim\sim\text{so}\sim\sim \text{ uniform } \sim\sim\text{that}\sim\sim \text{ just } g_{ave} = g_{cm} = g} = m \underset{\sim}{a}_{cm}$$ from p 3064 - rigid object

m a_cm → $\frac{dP}{dt}$ should've put it in on p 3063 - 3064 but I didn't

For a rocket, the the ~~explos~~ ejected burning fuel is ~~the~~ provides the external force on the rocket.

?070

Actually in this case the mass is actually being lost and you have to use the more general $\frac{d\cancel{P}}{dt} = F_{ext}$ since rocket mass changes.

$F_{ext} + mg = \frac{dP}{dt}$ which I won't bother to derive

~~In 1 dimension~~

Set to zero

$$\frac{dP}{dt} \quad \frac{dm}{dt} v + m\frac{dv}{dt} = v_{ex}\frac{dm}{dt}$$

$$\boxed{F_{ex} = v_{ex}\frac{dm}{dt}}$$

~~force~~ $-F_e$ must be exert on fuel to make it move at $-v_{ex}$ relative to rocket,

So F_e exerted on Rocket and force is reference frame invariant.

~~By conservation of momentum~~

~~$F_{ex} = v_{exhaust}$~~ $\frac{dm}{dt}$

~~rocket~~ parameter ~~from rest~~

~~Information~~

~~$F = (v_{ex} - v)\frac{dm}{dt}$~~

~~$(v_{ex} - v)$ is relative velocity ejected fuel in inertial reference frame~~

$$m\frac{dv}{dt} = (v_{ex} - v)\frac{dm}{dt}$$

$$\frac{dv}{v_{ex} - v} = \frac{dm}{m}$$

$$-\ln(v_{ex} - v)\Big|_{v_0 = 0}^{v} = \ln(m)\Big|_{m_0}^{m}$$

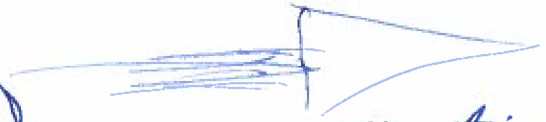
$$\ln\left(\frac{v_{ex} - v}{v_{ex}}\right) = \ln\left(\frac{m_0}{m}\right)$$

$$v = v_{ex} -$$

Let's set $mg = 0$, so

zero gravity or we measuring with respect to a local free-fall inertial frame. more likely

$F_{ext} = ?$



How momentum ~~change~~ ~~flow~~ creation is ~~time~~ frame independent ~~but not~~ force and by 3rd law the

Thus ~~the reactive force~~ force of ~~rocket on ejecta~~ ejecta on rocket

$$\therefore F_{ex} = -\left(-v_{ex}\left|\frac{dm}{dt}\right|\right)$$

$$= v_{ex}\left|\frac{dm}{dt}\right|$$

$$= -v_{ex}\frac{dm}{dt} \text{ since } \frac{dm}{dt} < 0$$

momentum ~~flow~~ creation from ejected fuel

$$= -v_{exhaust}\left|\frac{dm}{dt}\right|$$

$\frac{dm}{dt} < 0$

note 2 relative to rocket

m is mass of rocket

The mass m is of the rocket which decreases.

so

$$\frac{dp}{dt} = \frac{dm}{dt}v + m\frac{dv}{dt} = -v_{ex}\frac{dm}{dt}$$

$$m\frac{dv}{dt} = -(v+v_{ex})\frac{dm}{dt}$$

$$\frac{dv}{v+v_{ex}} = -\frac{dm}{m}$$

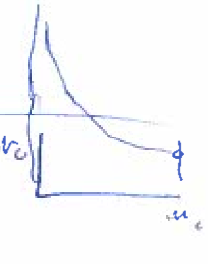
$$\ln\left(\frac{v+v_{ex}}{v_0+v_{ex}}\right) = \ln\left(\frac{m_0}{m}\right)$$

$$\therefore v = -v_{ex} + (v_0+v_{ex})\frac{m_0}{m}$$

$$v(m=m_0) = v_0 \text{ and } v(m\to 0) = \infty$$

Note this is a classical calculation and so $v(m\to 0)\to\infty$ is OK.

The whole ideal rocket use up.



3072)

3.1 Friedmann Equation

Derivation

Equation controls evolution of universe
→ via scale factor $a(t)$

cosmic time

We will derive semi-classically the way a 19th century physicist could have, but none did,

With some clairvoyance → which they could have had → after thought to generalize → feed back laws of GR to solve

The semi-classical derivation could've been done in the 19th century but actually was done in 1934 by Milne & McCrea (1934) (Wik) Bondi - 75

easum
static
univ.
galaxies
not
known
wrong
idea
on
inertial
frames

After the GR derivation of Friedmann 1922 (Wik) and, independently, LeMaître 1927s (Wik)

Wells (2014) gives a more rigorous demonstration why the GR and Newtonian derivations are equivalent (heavy going).

ut is
the
necessary?
ut
the semi
classical
derivation
did given
correct hypothesis?

a) First we assume all Free fall frames (unrotating with respect to whole universe) are true inertial frames without need of inertial forces

(19th century could have done this but didn't ⟹ the right substitute for Newton's wrong hypothesis of absolute space

absolute space always wrong hypothesis
— classical physics limit of Relativistic physics
⟹ true emergent Theory but not absolute space

b) Assume cosmological principle whole universe homogeneous and isotropic → can be only scaling up or down under gravity

on at rest

big enough space beyond boundary has effect

→ on large enough scale

3073

— ~~and all~~ or some other gravity like mass dependent force maybe

c) gravity & linear force only ones

The linear force ~~that~~ odd partner of inverse square law.

d) Assume you can approximate clumped matter by perfect fluid
homogeneous, isotropic, inviscid, but ~~maybe~~ you of Dar Eu
have pressure
(maybe a mystery pressure)

But this pressure is uniform overall.

e) Assume Shell theorem (Now we know GR Birkhoff thm) applies to a boundless system
— infinite flat (Euclidean)
— or with real imagination finite → a 3-sphere (a Euclidean sphere in 4-d Euclidean space, Wik topic
— or infinite hyperbolic space

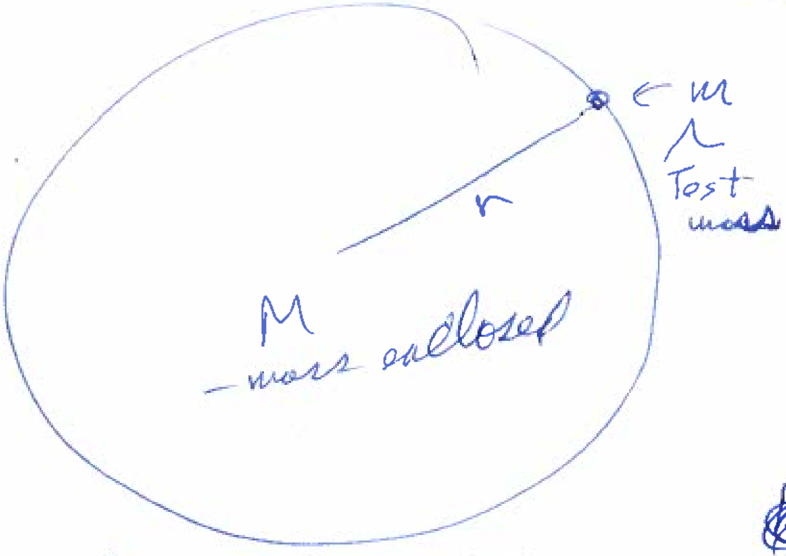
Closed if you extend but maybe

Actually one might limit this to a part of whole universe — maybe well beyond our observable universe there are other realms, boundaries but so far in 2023 — no observable signs — But maybe one day

But on small scale asymptotically flat as our 19th century physicist would know.

f) Newtonian physics holds ~~on a sr~~ asymptotically on a small and slow enough scale — because we know that is true — without the unnecessary hypothesis of Absolute space — with the alternative of all free-fall frames are true inertial frames.

g)



— sphere of radius r in our boundless space

— by shell theorem gravity and the ~~(by extra hypothesis~~

~~by~~ linear force

by analogue shell theorem with "renormalization" (see p. 3057-3060)

We say all beyond r has no effect

~~Consider test mass m and apply Work-Energy th~~

Our forces by shell theorems

$$F_g = -\frac{GMm}{r^2}\hat{r}$$

$$F_e = \frac{\Lambda}{3} m r \hat{r}$$

< by clairvoyance tells us the linear force should have this parameterization.

By integration

We assume M conserved for this integration

$$U_g = -\frac{GMm}{r} = -\frac{G\left(\boxed{\frac{4\pi}{3}}\right)\rho r^3 m}{r} = -\frac{4\pi}{3}G\rho r$$

$$U_e = -\frac{1}{2}\frac{\Lambda}{3} m r^2$$

Wik

$$\nabla \cdot f = \frac{\partial f}{\partial r}\hat{r} + \ldots$$

angular term

Now for another curious point of clairvoyance

Another curious symmetry between gravity and linear force — Written this way they both depend on r^2

$\rho \neq \rho(a$ But no party r!!

On p. 3076 we show they must or Dark matter interpretation all U must depend on

What if

$M = M(r)$?

That is mass is NOT conserved as r changes.

de since these really d conserv ma

Should we change U_g to reflect the fact that $F_g = -\frac{GM(r)}{r^2}$ now has another ~~mass~~ radius dependence?

Sort of clairvoyance again 4 Ar argument on p. 3076

The Answer is no, Our 19th century physicist doesn't know how U_g should change with NONconservation of mass and further developments suggest it's ~~[illegible]~~ Essential to say the U_g formula shouldn't change, Same gravitational Potential energy formula in this non-classical extension.

uniformity in inertial frames to which all physical laws referenced exept R

Note the creation/destruction of mass also (negative creation) creates it with kinetic to preserve homogeneity.

isotropy hmmm

But I'm not sure if this helps the hypothesis.

2016

degenerate ~~in class~~ with true classical physics.

9) Now apply conservation of mechanical energy

$$\underbrace{E_{mech}}_{\text{conserved}} = \frac{1}{2} m v^2 - \frac{4\pi G \rho r^2 m}{3} - \frac{1}{2}\frac{\Lambda m r^2}{3}$$

If not, you end up with ~~a~~ universal ~~pass~~ parameter controlling evolution of cosmos

To make formalism work nicely this term must depend on r^2 and not r^p explicitly

$\Rightarrow \rho$ has it's own $a(t)$ ~~radius dependence~~ scale factor dependence but cannot depend on r

Another perspective if there was non conservation of mass and at t_0 it stopped, then this formula should start applying at t_0 which is anytime ~~so that~~ So maybe it should apply at all times even when mass is <u>not</u> conserved.

Now ansatz $r = r_0 \, a(t)$

r_0: comoving coordinate at fiducial time t

particular value in a homogeneous universe

$a(t)$: dimensionless cosmic scale factor

Evolution of universe can't have the

$$\dot{r} = r_0 \dot{a}$$

and divide thru by $\frac{1}{2} m r_0^2 a^2$ $\llcorner (r_0 a)^2$

(if there were ~~some~~ $\Lambda \neq 0$ then test particle dependence free

$$\frac{E_{mech}}{\frac{1}{2} m r_0^2 a^2} = \left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi}{3} G\rho - \frac{\Lambda}{3}$$

(Liddle 55)

Therefore this side must be independent of test particle

Independent of test particle peculiarities.

Note if this term had depended ~~before~~ division here on r_0^p then r_0^{p-2} would be here and the RHS would have test particle dependence and we argue that cannot be.

The test particle and r_0 were arbitrary since our spherical cavity was general except sufficiently small to be in the classical limit,

Evolution of whole universe can't depend on test particle.

So $U_g = -\frac{4\pi}{3} G \rho r^2 m$ on p. 3005 was essential for consistency even if non conservation of mass,

But Λ could have the dependence. assu… not

good reason

To make progress

K can't have t dependence on arbitrary test particle would.

Now define

$$k = -\frac{2E_m}{m r_0^2} \quad (a)$$

$$[k] = \frac{E}{E/\left(\frac{L}{T}\right)^2} = T^2$$

The minus is so $k > 0$ is called positive curvature to ~~correspond~~ differential geometry conventions.

Or

$$kc^2 = -\frac{2E_m}{m r_0^2} \quad (b)$$

$$\left[k = \frac{2E_m}{mc^2 r_0^2}\right] = L^{-2}$$

Liddle-24 does it both ways but on p. 55 settles on (a) but (b) as L^{-2} is more closely related to geometrical understanding → lecture

This k is called curvature (Liddle-24)

Liddle p. 33
$k = 0$ flat
$k > 0$ spherical
$k < 0$ hyperbolic space

It is a basic parameter of our ~~space~~ universe model. → It can vary depending on initial conditions

– But only GR tells us!!!

So are G and Λ, but they apply to all ~~worlds~~ world models

3 ~~8~~ ~~8~~ 8 a)

Solutions monotonic
→ except certain special cases we'll ~~consider~~ consider later

1st order, nonlinear
→ so linear combinations of solutions are NOT solutions in general
But might be
in special cases or approximately

8 ⓥ Friedmann eqn. in One standard form

we
ee points
n spacetime
~~[illegible]~~
rived for
small
here.
0000
nuk.

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{k}{a^2}$$

Extend
~~nonlinear~~
→ so
linear
~~combination~~
of ~~solution~~
~~[illegible]~~

a) ✓ Note a DE → true at every point at every time → therefore throughout spacetime → and it solutions valid too
~~where [illegible]~~

even if we don't know the true general physics (which is GR) and just know our classical limit and extra hypotheses

No size scale limit

Consequence

b) Hubble parameter

– at $t = t_0$, our present
~~[illegible]~~ ~~[illegible]~~ $H = H_0$ the Hubble constant.

dr₀
~~eqn~~
– n
)

Recall $\cancel{r}$ = $r_0 a(t)$ general comoving distance and proper ~~now~~, at our time

$\dot r = r_0 \dot a$

and so $\frac{\dot a}{a} = \frac{\dot r}{r}$ or $\cancel{v} = Hr$

cy
ze.
ifferentially
true.
s this must
be true?
es that
~~[illegible]~~ ~~[illegible]~~
~~[illegible]~~
e
here
nullli'm
constancy
ize?
eccm
mr²
ert
alive
E.? a

Only ~~[illegible]~~
~~[illegible]~~
r → r
direct
observables
~~[illegible]~~
as $r \to 0$

Actually true for all r → proper distance.

v is called a recession velocity

velocity not relative inertial frames but between them.

So solution $a(t)$ is cosmic scale factor for whole univers.

free fall
a) inertial frames
b) homist
homogeneity
and
isotropy
everywhere
c) we
could
use U_g
form
even with
non conservation
of matter.

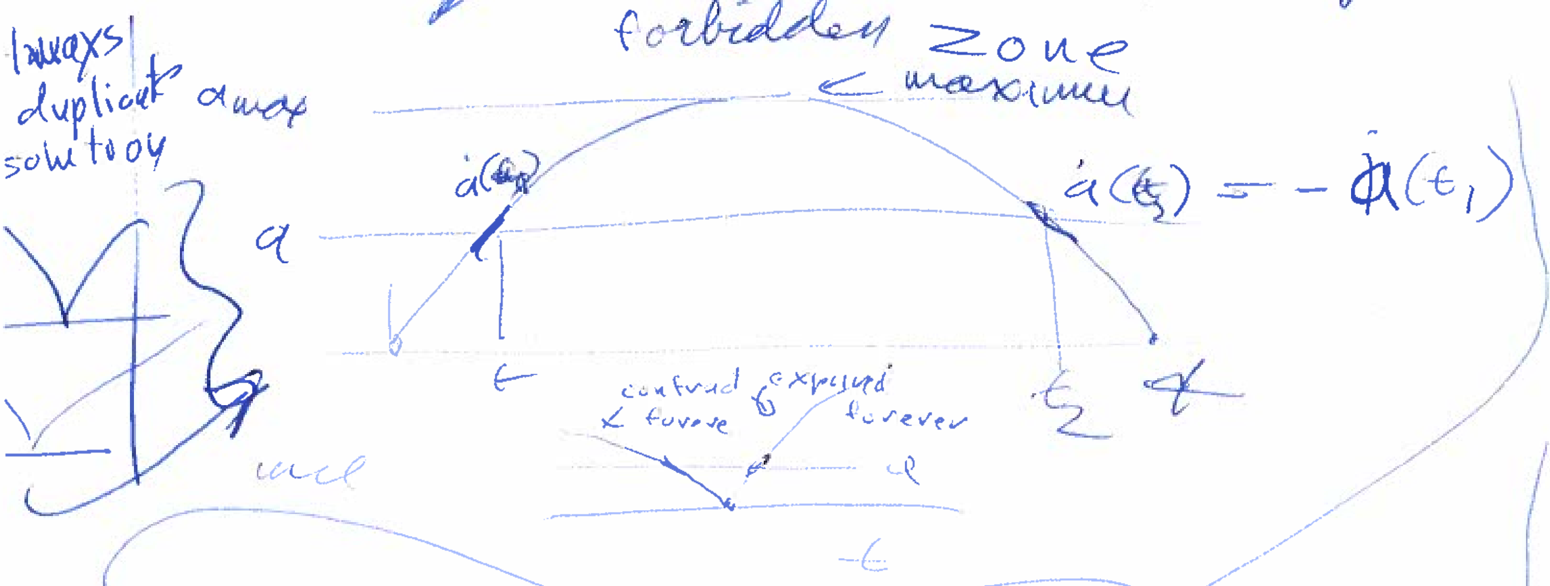
It can exceed the speed of light which would not bother our 19th century physicist at all

and not Relativity either since c is fastest physical speed relative to inertial frames and ~~[illegible]~~ is we limit between ~~[illegible]~~

v) Continue from p. 3080a

Note $\left(\frac{\dot{a}}{a}\right) = \pm\sqrt{\frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}}$

which sort of show you can have ~~stationary~~ points not at infinity

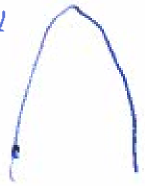


Under the radical is not like a driven, but more like a balance for motion

Analogous to $E_{mech} = \frac{1}{2}mv^2 + mgy$

KE PE

Trajectory of thrown ball



$v = \pm\sqrt{\frac{2E}{m} - mgy}$

To see a DE with a driven we need ~~the~~ acceleration equation (2nd Friedmann equation)

See p.3401

Difference of "sign" under radical traces back to derivation

2023sep10

3.1(still) The Friedmann Equation (FE)

(More on FE)

(derivation p.3072)

Most standard form

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

Hubble parameter - relative rate of expansion of universe

and cosmic time t same everywhere

i) True at at every point in spa and so they our Newtonian physics assump apply only on a small sphere

The FE uni and can be solved f a universe Derived fr GR by 19 Friedmann 19

known ~~My~~ ito in unit value

i) Non-linear → so two solutions do not add to make a third

iii) Autonomous → No ~~explicit~~ explicit dependence on indeptent variable t

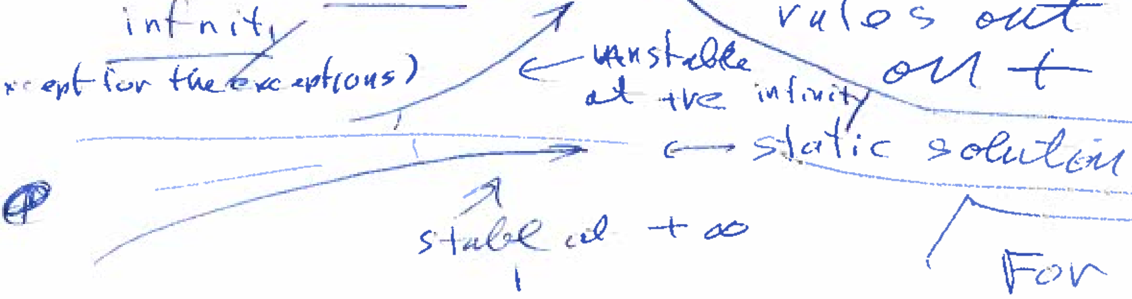
iv) 1st order

Unless you you hypothesize that G or Λ depend on t. Which we won't

I think the derivation rules out k depending on t

So Friedman-i equ often

Now 1st order Dif. equations can only have stationary points — max & Min at infinity (except for the exceptions)



Except for special cases of which FE is one

$$H = \pm\sqrt{}$$

the +/− cases allow a max or min

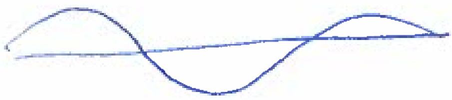
For FE the Einstein univ solution is static but unstable

perturbation (I think both + ~~to~~ & −v perturbation yes

show later

3080a)

$x = \sin t$
$x = \cos t$ ✓

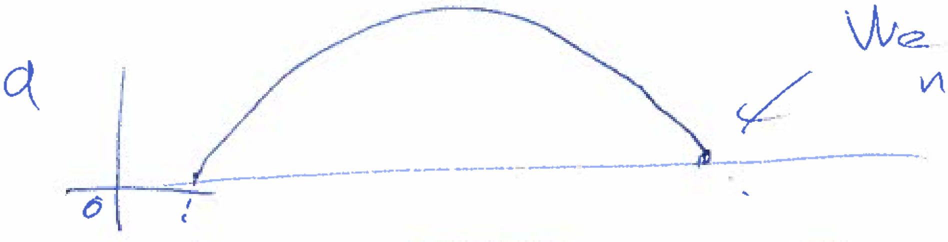


$$\dot{x} = \pm\sqrt{1 - x^2}$$

is another famous case → solution sine & cosine

In fact, the FE has sine/cosine solution in (conformal time) & we'll look at ~~which~~ it later.

but it can only have ½ cycle



We have no guidance at a = 0 points — how to integrate through

Can't add constant Einstein soln since FE is nonlinear

extra hypotheses allow cyclic universes — and there are all kinds from early days to now

v) See p. 3079a

vi) Recall $r = a(t) r_0$

~~Soln~~ for $dr = a(t)\, dr_0$

At one cosmic time for our derivation

$r = a r_0$ for finite since FE applies at all points in space.

$\dot{r} = \dot{a} r_0$

$\dot{r} = \frac{\dot{a}}{a} r$

$v = H r$

The general Hubble law applies at one instant in cosmic time t

Fiducial distance at $t = t_0$

So r is true physical distance = proper distance measurable at one instant in cosmic time

v is recession velocity.

Not a velocity relative to an inertial frame, but a velocity between inertial frames

It can and does exceed c.

c is ~~the~~ fastest physical speed relative ~~to an~~ inertial — ~~the~~ speed at which something goes past you right here, right now.

~~As know now~~ Lemaitre (1927) was probably first to explicitly put their equation in <u>print</u> though Friedmann probably knew it.

In fact, Lemaitre (1927) ~~also~~ derived ~~the~~ asymptotic form ~~then~~ ~~that~~ can be fitted to local measurement (O'Raifeartaigh 2019)

1st order Doppler $\Delta z = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$

1st order asymptotic as $z \to 0$

$$zc = H_0 r$$

(Cosmological redshift

$$f = \frac{L}{4\pi r^2}$$

$$r = \sqrt{\frac{L}{4\pi f}}$$

luminosity distance. — an observable, but not a proper distance except as $z \to 0$ asymptotically

because things move as light travels.

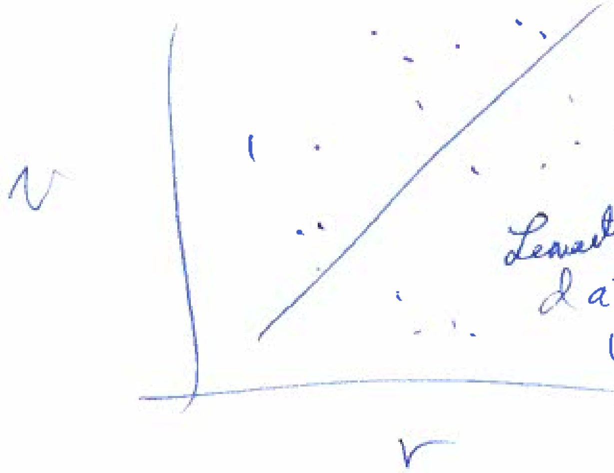
This is vaguely obvious if Doppler shift and cosmological redshift are conflated — which is only valid asymptotically as $z \to 0$ as we'll show later.

3080c

I've had a closer look at history of Lemaitre (1927) reception where he derived $zc = Hr$ and found $H_0 \simeq 600 \frac{km/s}{Mpc}$

— it was not ignored because it was in French. Lemaitre was well known to Eddington & de Sitter and talked to Einstein directly ~~to~~ in 1927 about his work

and sent them reprints to Eddington O'R 2009 p. 1

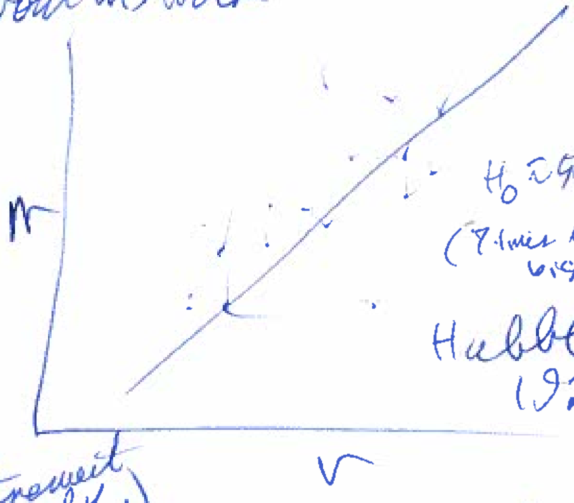


Lemaitre data 1927 from literature

— no improvement over already published data

↘ No clear linear relation or even expansion

↓ some like Einstein still favored static universe



$H_0 \simeq 500 \frac{km/s}{Mp}$ (7 times too big — due to systematic error)

Hubble 1929

Hubble's data was just a bit better and convinced people of the linear expansion & expansion in general

It seems, the Lemaitre 1927 paper just didn't convince people of expansion & linear expansion

But it does seem Eddington, de Sitter, other & Einstein were somehow obtuse

Lemaitre himself should've ~~sold~~ promoted more than he did

[3080 d

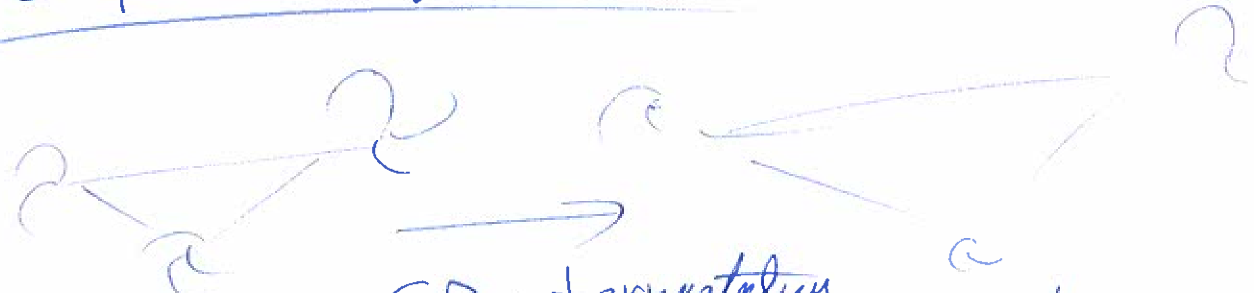
Everyone just seems to have not seen how important a derivation of general expansion and the asymptotic Hubble's law were — including Lemaitre

When Lemaitre published an English translation in MNRAS in 1931 he dropped the discussion of Hubble law & confused of ~~the~~ actual interest

(he meant current & since what actual means en Francais)

didn't want to get into a priority fight with Hubble then and was much more interested in promoting his Primeval atom origin of Lemaître universe (as we ~~now~~ call it)

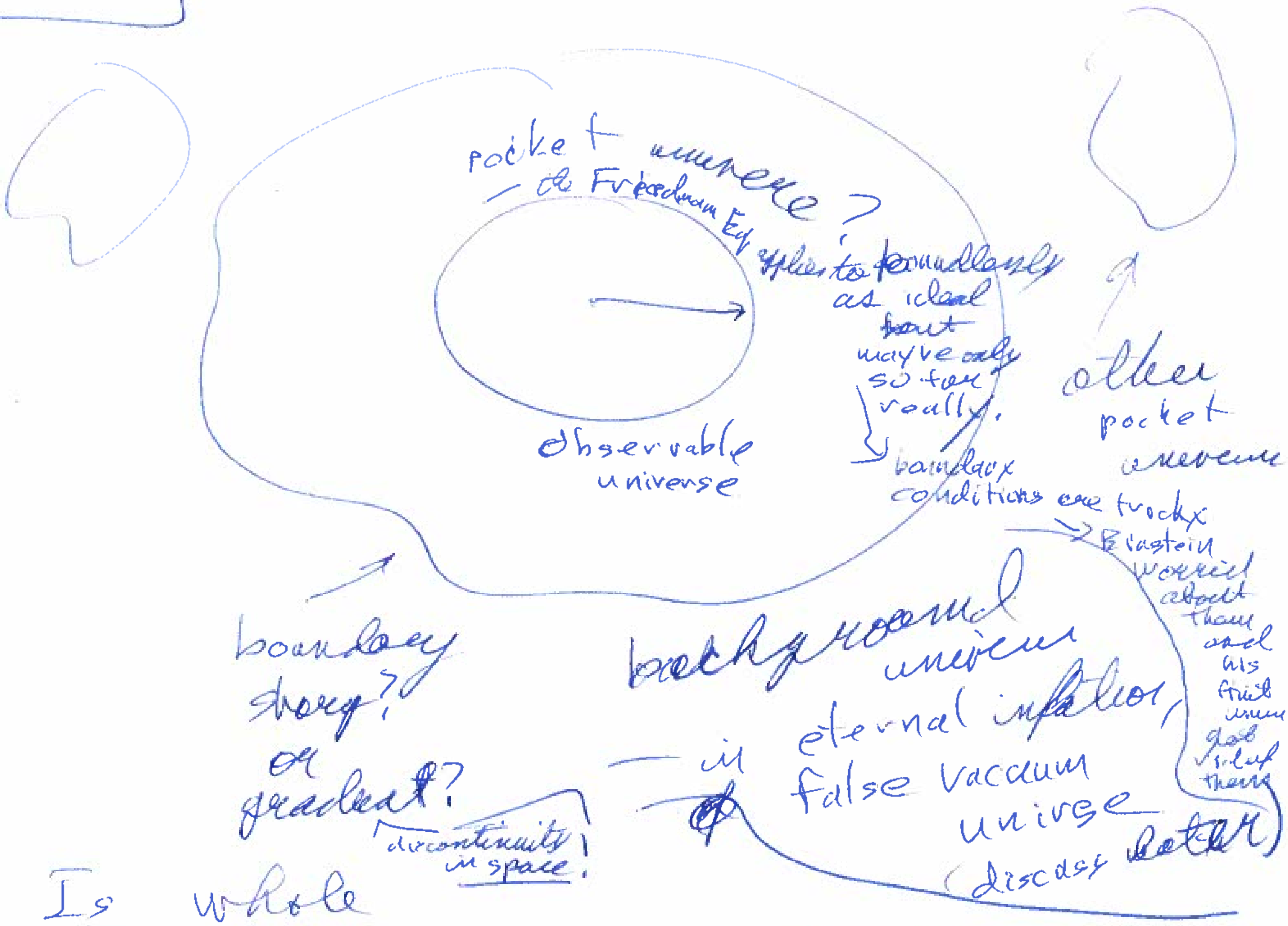
vi) Expansion of Universe



Newtonian interpretation wouldn't have included growth of space but otherwise the same

GR interpretation growth of space between bound systems (in a sense points which center on participating in mean expansion of universe → Center of mass of basic inertial frames comoving frames

All free-fall frames — the most ~~basic~~ ones free fall in the ~~[illegible]~~ [illegible] universe at lea



Is whole universe in expansion or contraction or some mixture.

If GR applies universally, no stable static cases.

2023sep10

d) Scaling the Friedmann Equation (FE)

$$H^2 = \left(\frac{\frac{da}{dt}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a} + \frac{\Lambda}{3}$$

Most standard form but for analytic & maybe numerical solutions one often scales

(or kc or on ~~[illegible]~~ p. 3077)

$\Lambda = 8\pi$

$$H^2 = \frac{8\pi G}{3}\left[\rho \underbrace{- \frac{3k}{8\pi G a^2}}_{\equiv \rho_k < 0} + \underbrace{\frac{\Lambda}{8\pi G}}_{\equiv \rho_\Lambda}\right]$$

$\equiv \rho_k < 0$ for $k>0$ +ve curvature, $\rho_k = 0$ for $k=0$ zero curvature, $\rho_k > 0$ for $k<0$ -ve curvature

The sign switch is a nuisance.

$\equiv \rho_\Lambda$ (Lambda density)

dark energy

- if really Λ then no really energy at all.
- if constant dark energy simplest real ener
- ~~if~~ dynamic dark ene beyond our scope but ma be tr

Now define a fiducial time t_0 and scale factor $a(t_0) = a_0$.

t_0 is often cosmic present and $a_0 = 1$ often, but neither have to be.

We define $H_0 = H(t_0)$.

Then at t_0

$h = \frac{H}{H_0}$, $H_0^2 = \frac{8\pi G}{3}\rho_{\text{total }0}$

Hubble density H_0

H_0 and $\rho_{\text{crit }0}$ are alternative parameters

Hubble Collective ρ_{total} — analogous to Hubble Hubble

So if you know H_0 (set it or empirically),

Either H_0 or ρ_{crit} can be a free parameter

$$\rho_{\text{H}_0} = \frac{3H_0^2}{8\pi G} = \rho_{\text{critical},0}$$

{Always called the critical density

Or alternatively $\rho_{\text{total }0}$ sets $H_0^2 = \frac{8\pi G}{3}\rho_{\text{total }0}$

Note if the observed H_0 is known itself ρ_{crit}, the ~~and~~ ρ + [illegible] = ρ_{crit}

then $\rho_k = 0$ and the observable universe is fla

3082

if $\rho + \rho_\Lambda < \rho_{crit}$ then $\rho_k > 0$ and universe has -ve curvature | $\rho + \rho_\Lambda > \rho_{crit}$, $\rho_k < 0$ and universe has +ve curvature

- universe model
- may not extend to boundlessly.

To complete scaling

$$t_{H_0} \equiv H_0^{-1} \text{ Hubble time } = \frac{1}{\sqrt{\left(\frac{8\pi G}{3}\right)\rho_{crit}}}$$

scaled time $\tau = \frac{t}{t_{H_0}}$

which is a characteristic time. It is only the age of the universe in one special case as we'll see

and scale $x = \frac{a}{a_0}$

so $x_0 = 1$ at $t = t_0$, $\tau = 1$
$\tau_0 = t_0/t_{H_0}$

Of course a_0 is often 1, but it doesn't have to be and I find it hard to think of "a" as a variable.

Define $\Omega_p = \frac{\rho_p}{\rho_{crit}}$ a density parameter for density component p

Define $h \equiv \frac{H}{H_0}$.

Then FE becomes

$$h^2 = \left(\frac{\frac{dx}{d\tau}}{x}\right)^2 = \underbrace{\Omega}_{\text{total density parameter — or Hubble density parameter}} = \sum_p \Omega_p = \sum_p \Omega_{p_0} x^{-p}$$

If we assume only power-law components which we do ~~but~~ one can be more general.

at $x = x_0 = 1$

$$h_0^2 = 1 = \Omega_0$$

Note you can set all ρ_{p_0} and then they determine ρ_{crit0} and H_0.

Or you can set H_0 and that sets ρ_{crit0} and then one of the ρ_{p_0} is determined which in practice is ρ_{K0} or if only one component, it is that ρ_{p0}

istrau
et al.
2023
rom
re-analysis
Planck-2018
ine
$_K = -.012(10)$
o
onsistent
th ⊙
ithin nearly
1σ.
k<0 is
+ve curvature

There are five Powers P's widely considered, but there could be others and also non-power law dependence on x ~~or~~ maybe explicitly on time t or τ

3083

$P = 0$, is for dark energy of Λ type — true cosmologica constant or true constant dark energ

$P = 1$ for quintessence in some theories

$P = 2$ for curvature or cosmic strings in some theories Ω

$+k$, s
$-k$, 2

or $R_h = ct$ universe of Fulvio Melia (e.g. 2014)

(It's disfavored by most, but Fulvio does keep getting papers on it accepted — his Bayesian analyses keeps favoring it which suggests there's a lot of art in Bayesian analysis

se
r
Evid
z
to re
dec

$P = 3$ for "matter" = "dust" = matter at rest in comoving frames which approximates ~~all~~ almost all matter in observable universe which is almost all NRn-relativistic (NR) matter in comoving-frames and CDM

Baryonic & CDM

baryonic matter

$P \propto \frac{1}{a^3}$ an ordinary conserved matter in expanding space

We'll prove this for Radiation in Lecture 5

$P = 4$ for "radiation" = all extreme relativistic photons and cosmic neutrinos up to some time — they slowed to NR at some

We'll discuss cosmological redshift & slowing in lect 5

3084

early time (t ≃ 10^6 Myr by some theory I think — it's depends on their mass (unknown)

but their relativistic early phase is important in modeling early universe.

iggeser Powery
Faster falloff.
rad $^{r=4}$ must
be less
matter $r=3$
m than $r=2$ if any
ess than $r=1$ if any
than $r=0$ eventually

) First Glance at Solutions of FE

So Scaled Friedmann Eq.

$$\left(\frac{\frac{dx}{d\tau}}{x}\right)^2 = \sum_p \Omega_{r_0} x^{-p}$$

- 1st order
- non linear
- ordinary DE

Not 2nd order as we often think of eq. of motion

be
uem
universe
phases

Mult by x^4, $r_{m}=4$. $d\eta = \frac{d\tau}{x} = \frac{dx}{x^2\sqrt{}} = \frac{dx}{\sqrt{\Sigma_r x^{4-p}}}$

~~Also~~ Can this be solved analytically for all of $p = 0, 1, 2, 3, 4$?

Σ density component solution

Sort of but mostly not.

Frank Steiner 2008 in an conf. paper only found on his own website (not arXiv)

An applied mathematician who does a little cosmology — an old Herr Professor who knows all special functions

shows $a(\eta) =$ analytic solution (p. 9)

in terms of the Weierstrass Elliptic (p. 7)
℘-function

η is conformal time

$$t(\eta) = \int_0^\eta a(\eta)\,d\eta$$

Steiner p. 3
Je 23 p. 9

which can only be solved ~~analy~~ numerically.

> a special function and so only analytic by definition

⟷ need numerical tabulation
→ or fast numerical evaluation

If there is only one power component, analytic solution are easy as we'll show in a moment.

If there are only two power components simple analytic solutions are always obtainable $x = x(\hat{\eta})$

$\underbrace{\qquad\qquad\qquad}$ as function of generalized conformal time

All this may be my own discovery, but it may known buried in literature somewhere ~~Maybe~~ Willem de Sitter 1930 who worked out a lot analytic and easily solved numerical solutions

~~One~~ Two components $p \cancel{\geq} q \geq 0$

$\Omega = \Omega_{p0} x^{-p} + \Omega_{q0} x^{-q}$

~~W~~when $Q = \frac{q}{p-q}$ for integer

No analytic solution for Q non-integer > 0

$Q = 0$ is an integer

(Je23, p.22)

in the sense $\left\{ \begin{array}{l} x = x(\hat{\eta}) \\ \tau = \tau(\hat{\eta}) \end{array} \right.$ there are both analytic

and if $Q = 0, 1, 3$ then $x(\tau)$ ~~can be~~ and $\tau(x$ can be ~~often~~ obtained explicitly (Je23 p. 22)

$p = \left(\frac{Q+1}{Q}\right) q = \left(1 + \frac{1}{N}\right) q$ for N integer

We'll look at interest cases in lecture

There are analytic solution for 3 power components in special cases: ~~$p > q >$~~

$\Omega_p x^{-p} + \Omega_q x^{-q} + \Omega_0 x^{-r}$ p, q, r

$p \leq 2q$ and $r = 0$

(Je23, 43-44)

3086

Numerical solution for $\tau = \tau(x)$ are easy

$$d\tau = \frac{dx}{x\sqrt{\sum_p \Omega_{p0}\, x^{-p}}}$$

$$du = \frac{d\tau}{x} = \frac{dx}{\sqrt{\sum_p \Omega_{p0}\, x^{2-p}}}$$

or any other density form you like.

But analytic solution are still vital for

a) understanding.

b) checking numerical techniques

c) dealing the vast scale difference between the early radiation phase and the later matter & Lambda phases of universe as we'll discuss later.

As we'll see $x(\tau)$ rad-matter & $x(\tau)$ matter-Λ have exact analytic solutions, but not rad-matter-Λ (no fun)

f) A Second Glance ~~of~~ at solutions

Only one power component and $p \geq 0$. $p=0$ is the pure cosmological constant universe

They all have to be flat universes

This means $\Omega_{p0}=1$ actually and t_0 or [illegible] $=\Gamma_{p0}$ are alternative parameters

$$d\tau = \frac{dx}{\sqrt{\Omega_{p0}}\, x^{-\frac{p}{2}+1}} = \frac{x^{p/2-1}}{\sqrt{\Omega_{p0}}}\, dx$$

We don't worry about $p<0$

one power early time leading power!!

$$\tau'\Big|_0^{\tau} = \frac{1}{\sqrt{\Omega_{p0}}} \frac{x^{\frac{p}{2}}}{\frac{p}{2}}\Big|_0^{x}$$

no sense considering non-zero start points

Point origin or Big Bang singularity

$$\tau = \frac{1}{\sqrt{\Omega_{p0}}} \frac{2}{p} x^{p/2}$$

or inverting

$$x = \left[\sqrt{\Omega_{p,0}}\, \frac{p}{2}\, \tau\right]^{2/p}$$

in general

Recall $\Omega_{p0} = 1$ in all cases p I just like leaving Ω_{p0} in in order to see where it is and

These are $\tau \to 0$ cases where each $\Omega_{p,0}$ is ~~[illegible]~~ the leading small terms

$$= \left[\sqrt{\Omega_{p=1,0}}\, \frac{1}{2}\, \tau\right]^{2}$$

for p = 1 quintessence

$\Omega_{p=2,0} = 1$ and would

$$= \left[\sqrt{\Omega_{p=2,0}}\, \frac{2}{2}\, \tau\right]$$

for $R_u = ct$ universe

or cosmic string

Just a linear scale up.

I think can a just curve

This is the case where τ_0 is the age since point origin

$$= \left[\sqrt{\Omega_{p=3,0}}\, \frac{3}{2}\, \tau\right]^{2/3}$$

for matter universe

$t \gtrsim 50\,\text{kyr}$

$t \lesssim 10\,\text{Gyr}$ Cahill p. 5, 9.

$t \gtrsim 10\,\text{Gyr}$ is Λ dominated era

The Einstein 4-d's u~~niverse~~s (see Below)

$t \lesssim 50\,\text{kyr}$ Cahill p. 5

$$= \left[\sqrt{\Omega_{p=4,0}}\, \frac{4}{2}\, \tau\right]^{\frac{1}{2}}$$

for radiation ~~universe~~ era universe

$\Omega_{\mathrm{rad\,CMB},0}$

All these solutions have point origins AKA Big Bang Singularity

3088

(EdS)

9) The Einstein-de Sitter Universe (1932)

Eds phase 50 kyr - 10 Gyr mater dominated EdS phase

Which is not the static Einstein Universe (1917) That we'll consider in Lecture 5

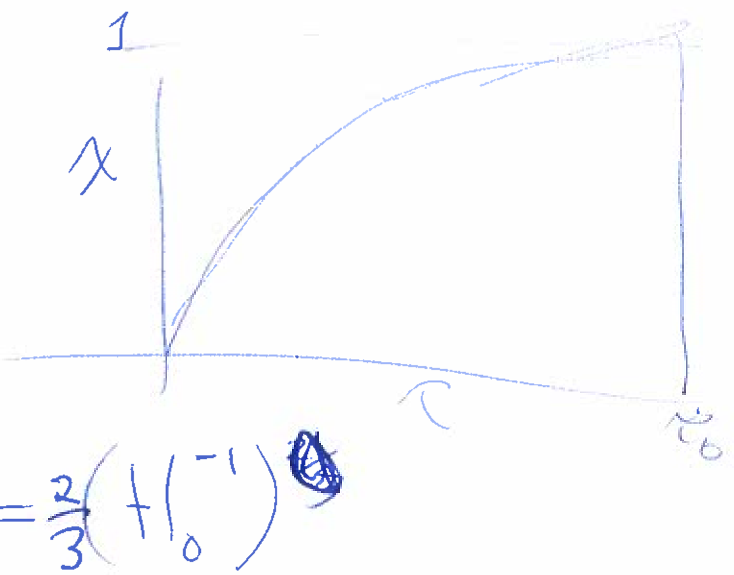
Nor the de Sitter Universe (1917) which we'll consider in a moment

So a pure matter universe

and a flat infinite universe

$\therefore \Omega_{3,0} x^{-3}$ is the only density and $\Omega_{3,0} = 1$

$$x = \left(\frac{3}{2}\tau\right)^{2/3}$$



$$x = x_0 = 1 = \left(\frac{3}{2}\tau_0\right)^{2/3}$$

$$\therefore \left(\frac{3}{2}\frac{t_0}{t_{H_0}}\right)^{2/3} = 1$$

$$t_0 = \frac{2}{3} t_{H_0} = \frac{2}{3}\left(H_0^{-1}\right)$$

In general $t_0 = \frac{2}{p}\left(\frac{13.968}{h_{70}}\right)$ (Je23-92)

$h_{70} = \frac{H}{70}$

p=3 for EdS

In 1932, $H_0 \doteq 500$ ↪ p=2 for EdS

$t_{0_{EdS}} = \frac{2}{3} \cdot 2$ Gyr = 1.3 Gyr

which was a problem EdS

did not worry about.

$t_{age\,E} \simeq 3Gyr$ in 1930 from radioactive dating

Actual EdS paper is very short and its main point was ~~the~~ Hubble density (critical dens formula

$$\rho_{H_0} = \rho_{crit\,0} = \frac{3H_0^2}{8\pi G}$$

there computed density was of order the estimate ~~that~~ the $\rho_{observed}$

and so a flat. infinite universe model was what ~~they~~ suggested.

The idea was the simplest model allowing expansion.

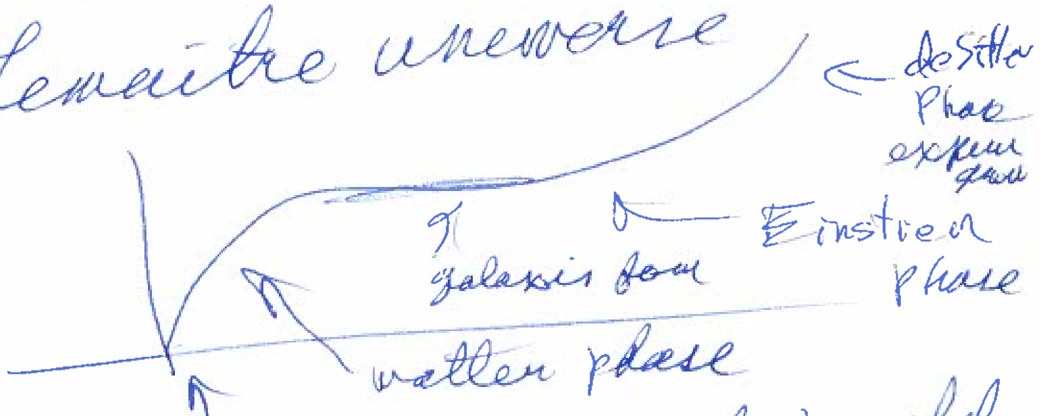
In 1933 review article, Einstein made clear he thought tracing the model back to early times was premature and so ~~the~~ age problem did not worry him

3090

Actually the EdS model was not adopted as the ~~accepted mod~~ standard model until the 1960s it seems,

Bondi 1960 in his book on cosmology supports several models were considered.

– Lemaître universe



origin → Lemaître's cold big bang primeval atom was mostly ruled out

→ but hot big bang was consider

Bondi, Hoyle and Gold favored steady state universe → which is sort of the de Sitter univer.

Bondi may have given up on it by 1960, but he doesn't say that explicitly

3091

In the 1960s, the evolution of ~~the~~ universe from quasars was accepte (1963 discovery)

radio source counts (more in the past than now)

and CMB (1965) strongly supported (not) Big bang & Big Bang Nucleosynthe to H, He, Li

at that point the EdS universe become standard Model of cosmology

$t_{H_0} = H_0^{-1}$

=

$H_0 = 50 — 100$ in those days ~1960-199

so $t_{0_{EdS}} = \frac{2}{3}\left(\frac{13.96\,\mathrm{Gyr}}{h_{70}}\right)$

$= \frac{2}{3} t_{H_0}$

$\frac{14}{15} \cdot 14 \approx \frac{196}{15}$

$= 13\,\mathrm{Gyr}$

for $H_0 =$

$\frac{14}{30} \cdot 14 \approx 7\,\mathrm{G}$

$H_0 = 100$

but allowed Sandage + Tammann

far too short for $t_{\text{age Globular Cluster}} \gtrsim 13\,\mathrm{G}$

$\rho_{H_0} = \rho_{crit} = 8{,}523 \times 10^{-27} \frac{kg}{m^3} h_{70}^2$

$= 1{,}259 \times 10^{11} \frac{M_\odot}{Mpc^3} h_{70}^2$

$\rho_{stars} + \rho_{dark}$ (baryonic matter) was of this order

G. de Vaucouleurs

— and the EdS model was still allowed,

However by the 1990s $\Omega_{stars} + \Omega_{dark} \approx 0{,}3$

which in fact what they still are.

which implied $\Omega_k = +.7 > 0$

or a strongly ~~positively~~ negatively curved universe.

but there seemed no other indication of this in the 1990s

and inflation (1979) said $\Omega_k \simeq 0$, but not exactly.

Tristram et al 2023 $\Omega_k = -.012 \pm .010$

Furthermore, $H_0 \rightarrow 70$

which gave $t_{0\,EdS} = 9$ Gyr

$t_{0\,EdS} = \frac{2}{3}\left(\frac{13.968}{h_{70}}\right)$ too short for Globular cluster age

So, in fact,

c. 1995 before the discovery of the acceleration of universe

$t_0 = \frac{13.963 \text{ Gyr}}{h_{70}} \cong 14$ Gyr

what we now call the Λ-CDM began to be considered (Scott 2018 p.10) history of cosmology

which is still is

$\Omega_\Lambda \cong .7$ (see no need for $\Omega_K \cong .7$)

which gave $\Omega_{total} \simeq 1$

and flatness and acceleration (as we'll show later) ~~$t_0 \simeq 14$ Gyr~~ ~~Hubble~~

ΛCDM gives $t_0 = 13.8$ Gyr coincidently close to Hubble time

So 1995 — 2015
the Λ-CDM ruled
— fit the acceleration
— the age constraints from Globular clusters,
— the CMB fluctuation with
Dark matter $\Omega_{\text{Dark}} \approx 26$, $\sim \Omega_{\text{Baryon}} = .0$
gave the large-scale structure
in a Λ driven universe

And the Λ-CDM has defeated
several tension
e.g. — missing satellites problem
— planes of satellites problem → In Lect. ?

But the Hubble tension (already discussed)
& S_8 tension seem harder to beat.
(consider later briefly)

But this may go away too according to a recent paper

So it seems almost every week
at least one paper says
Λ-CDM is disfavored
~~but other~~ but at least one more
says it is still viable
or favored

094)

clearly it depends on what data set you are analyzing and how you do your Bayesian analysis and the fine art of choosing priors.

h) The de Sitter Universe (1917)

— modern version which is not what de Sitter found exactly) simplified version — cosmological redshift

$$P = 0$$

$$\therefore \quad d\tau = \frac{x^{-1}}{\sqrt{\Omega_\Lambda}}\,dx \quad (\text{see p. } 3086)$$

$_0^2 = \frac{\Lambda}{3}$ (see p. 3081)

H_0

$H_0 = \sqrt{\Lambda/3}$

$H_0 = \sqrt{\Lambda/3}$

$$\tau\Big|_{\tau_0}^{\tau} = \frac{1}{\sqrt{\Omega_\Lambda}} \ln x\Big|_{x_0}^{x} + ~~\text{const}~~$$

$\longrightarrow = 1$ recall

$$x = x_0 e^{\tau - \tau_0}$$

$\tau_0 = \frac{t_0}{t_{H_0}}$ (p. 3082)

So no point origin and so choose $t_0 = 0$

$$x = x_0 e^{\tau} = x_0^{H_0 t} = x_0 e^{t/t_{H_0}}$$

$$x = e^{(\sqrt{\Lambda/3})t}$$

So the pure de-Sitter universe is eternal and exponential and only has dark energy in it.

The Λ-CDM model is asymptotically approaching this.

We examine the $a(t)$ for the Λ-CDM in ~~less~~ lot some detail

i) What of the Steady-State universe

— Vogue 1948 — ~ early 1960s

It posited an infinite eternal universe in which the dark energy was just ~~matter~~ ~~or~~ maybe dark matter ~~or~~ nonponderous matter

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}$$ is constant as universe expands.

It's spontaneously created at low density ~~probably~~ maybe as $H \Rightarrow \gamma^+ + e^-$ and maybe dark matter in a modern version.

Hoyle 1948
Bondi & Gold 1948
all at Cambridge and worked together but published separately
— actually in unpublished work 1931 Einstein briefly considered ~~something~~ ~~they~~ steady-state universe ~~too~~ but gave up on the idea since GR didn't naturally imply matter creation.

Con
~~the~~
chan

3096

old galaxy

- gravitationally evaporate eventually
— so no infinitely old ones
but space laced with isolated compact remnants

old galaxies refreshed by new
+ and maybe some dispersed if they got too old?
→ or maybe just fade out to invisibility — no idea of ~~what~~ happened to infinitely old galaxies was coined in the heyday of steady state theory in the 1940s

new galaxies ~~[illegible]~~ formed between old ones

ha|f way
with
stellar
nucleosynthesis
o BBN
ust
gives
, He, Li
ital
for all
EVs
203C

The Steady-State Model was a good theory — falsifiable — it predicted eternal unchanging universe with no place for CMB — but evidence from quasars (also radio galaxies) + CMB falsified it in 1960s

But Fred Hoyle (1914—2001) never gave up on some version of it, it seems. He became a bit of crank in later years

i) Density Variation for one-component Models with $p>0$

$$\Omega_p = \Omega_{p0}\, x^{-p}$$

see p. 3087

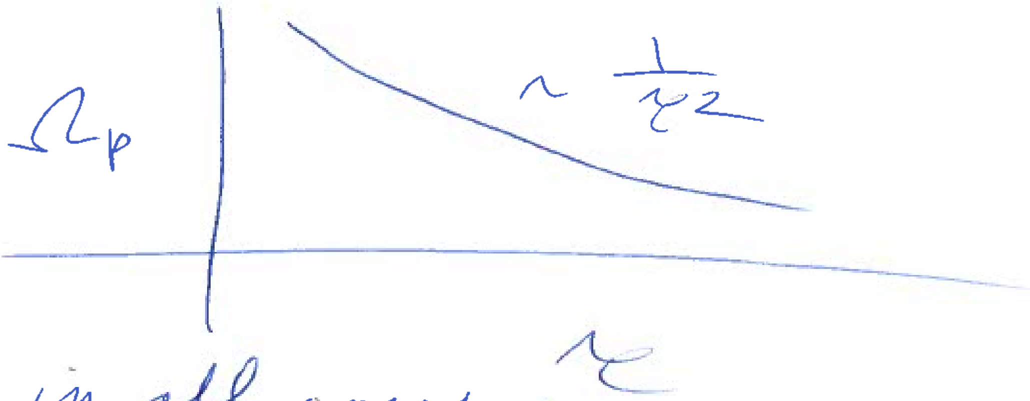
where $\Omega_{p0} = 1$

except $p=0$

$$x = \left[\sqrt{\Omega_{p0}}\, \frac{p}{2}\, \tau\right]^{2/p}$$

$$\Omega_p = \left[\left(\frac{p}{2}\tau\right)^{2/p}\right]^{-p}$$

$$\Omega_p = \left(\frac{p}{2}\right)^{-p}\tau^{-2}$$



in all cases for $p > 0$ ~~for~~

For $p=0$, $\Omega_{p=0} = \Omega_{p,0} = 1$

the density is constant of course.

3.4 Fluid Equation

What are the thermodynamics of ρ?

What is its equation of state (EOS)?

$$P = P(\rho)$$

T implies heat energy which implies mass and so in a vague way can be subsumed in P if one knows ~~[illegible]~~ how.

3098

Recall 1st law of Thermodynamics
and apply to our isotropic, homogenous universe.

$$dE = TdS - \sum_i P_i dV + \sum \mu_i dN_i$$

Temperature

entropy

Pressure types

$$P_i = P_i(\rho_i)$$

The pressure types may not be coupled.
The famous dark energy ~~matter~~ -ve pressure seems to pull on nothing except the dark energy stuff and if uniform cancels out a ~~simplest~~ assumption

and after decoupling era (~380kyr) - the CBR does couple to anything else either much - just flows through space

particles created or destroyed

Therefore suppress this term, but maybe someone knows what to do with it.

$$E = \rho c^2 V$$

volume

density - including thermal energy if need be.

→ so particle creation and its thermal energy can be subsumed here

We assume adiabatic since no outside for energy flows
(— but maybe there really is an outside but still not much energy flow)

$\therefore dS = 0$

$$dE = -\sum_i P_i dV$$

$$c^2(Vd\rho + \rho dV) = -\sum_i P_i dV$$

$$\therefore \ d\rho = -\left(\rho + \sum_i \frac{P_i}{c^2}\right)\frac{dV}{V}$$ Fluid equation

$$d\rho = -3\left(\rho + \frac{\sum P_i}{c^2}\right)\frac{da}{a}$$

$V \propto a^3$

$dV \propto 3a^2\,da$

$$\frac{dV}{V} = 3\frac{da}{a}$$

We need to make cosmological assumption about EOS. Simplest is

$P = w\rho c^2$ where w is a constant — just called w parameter of the ~~Cosmic~~ EO (Wik: EOS (cosmology))

$w = $

- 0 dust → ie. NR (Rest mass dominated) matter, baryonic or CDM
  - rest mass energy has zero pressure
  - stars galaxies ⊥ actual dust
  - dust is cosmically insignificant now
- $1/3$ "Radiation" → real Radiation or ER particles like cosmic neutrinos in early universe — They slowed down to be NR particles ~~because it~~ eventually because they have rest mass
  - (We will prove this later)
- $-1/3$ F. Melia's $R_h = ct$ universe stuff
- -1 dark energy (stuff)

~~$\frac{d\rho}{\rho} = -(1 + \sum_i w_i X_i)\left(3\frac{da}{a}\right)$ where $X_i = \frac{\rho_i}{\rho}$~~

~~$\ln\rho = -3(1 + \sum_i w_i X_i)\ln a$~~

~~$\rho = \rho_0\, a^{-3(1+\sum_i w_i X_i)}$~~

3000 a

(2023sep10)

~~ρ_0/a^3 $W_d = 0$~~ dw

~~Let $\rho = \sum_i \rho_i$~~

~~Then $\sum_i d\rho_i = -\sum_i \rho_i (1 + w_i)\left(\frac{3da}{a}\right)$~~

Posit All density types are independent

∴ $d\rho_i = -\rho_i (1 + w_i) \frac{3da}{a}$

$\ln \rho_i = -(1 + w_i) \frac{3da}{a}$

$\rho_i = \rho_{i0} \left(\frac{a}{a_0}\right)^{+3(1+w_i)}$

Just what you'd think

$\rho_{dark0} \left(a_0/a\right)^{+3}$ $w = 0$

$\rho_{rad} \left(a_0/a\right)^{+4}$ $w = 1/3$

~~one~~ per species cosmolog. redshift

$\rho_{Rct} \left(a_0/a\right)^{+2}$ $w = -1/3$

$w = -1/3$ needed to get this dependence

ρ_Λ $w = -1$

constant density

See p.3081 $\frac{\Lambda}{3}$ replace by $\frac{8\pi G\rho_\Lambda}{3}$

Negative pressure stuff

such ~~as it sucks in density on~~

+ve pressure loses energy to work on something → where to?

-ve pressure gains energy from work by something on it → but where from?

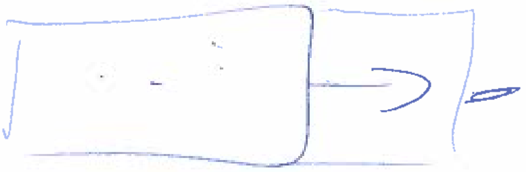
2019 jul 8

~~3096~~
31006

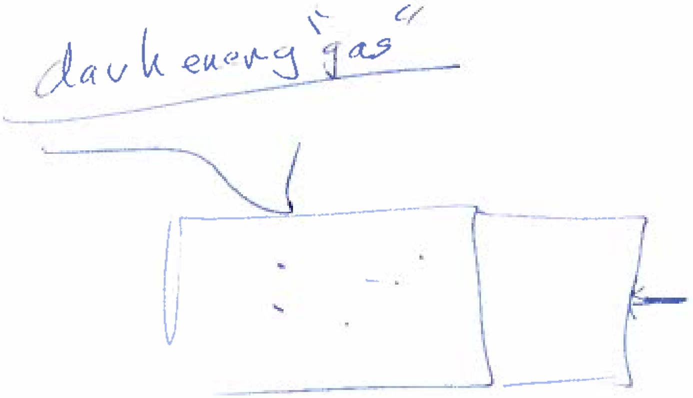
What does -ve pressure mean?

What is this stuff with -ve pressure?

Well in simplest concept, not much.



expand a normal gas adiabatically and +ve pressure causes work to be done by gas and it looses energy



expand a -ve pressure gas adiabatically and you have to pull ~~it out~~ on it and put energy in.

So from somewhere or nowhere dark energy flows into universe,

or energy flows into mystery darkenergy gas

~~Actually Dark energy~~ is what ~~Dark~~ the Negative pressure stuff has

~~bes~~ xpand univers

And the negative pressure is magically assumed uniform everywhere and has no gradients at all and pushes or pulls on nothing except itself (dark energy) (cancels out)

Simplest concept

Put into $P = -\rho_\Lambda c^2$

the dark energy ~~that~~ flows in is also absolutely uniform in simplest conception → ~~though~~ it has ~~gravity and in all con~~ gravitational and inertial mass — but somehow never clumps.

Created with "conversion" from KE

maybe it's self ~~pressure~~ is so strong resists of gravity? and ~~[illegible]~~ clumping

~~3066b~~
3000y

— by the way wherever created it has the "kinetic energy" in the sense of our Friedmann eq. consistent with that location (That relative KE PE)

No choice if homogeneity and isotropy to be preserved

the created mass has consistent relative KE in the sense of keeping

$$E_m = \frac{1}{2} m v^2 - \frac{4\pi G M \rho r^2}{3} + \frac{1}{2}\frac{\Lambda}{3} m r^2$$

balance equation constant

I guess what I mean is that curvature K stays constant | Must of our test particle not to control universe ?

Quantum Field theorists ~~on~~ on some general grounds ~~think~~ vaccuum ~~energy~~ energy ~~(or vac~~ (or vaccum) should have ~~this~~ behavior
→ but their simplest estimate is 10^{120} times too large for observation (W.k: Dark energy: Cosmological ideas)

i.e. for Λ dark energy

one fix is the Anthropic Principle
→ the vacuum ~~energy~~ energy or ~~[illegible]~~ Λ is somehow stochastically chosen in pre-inflation ~~and~~ we would not be here if it were too big → matter would be expanded so much that no clumping into ~~[illegible]~~ galaxies, beings

If -ve Λ somehow, premature collapse.

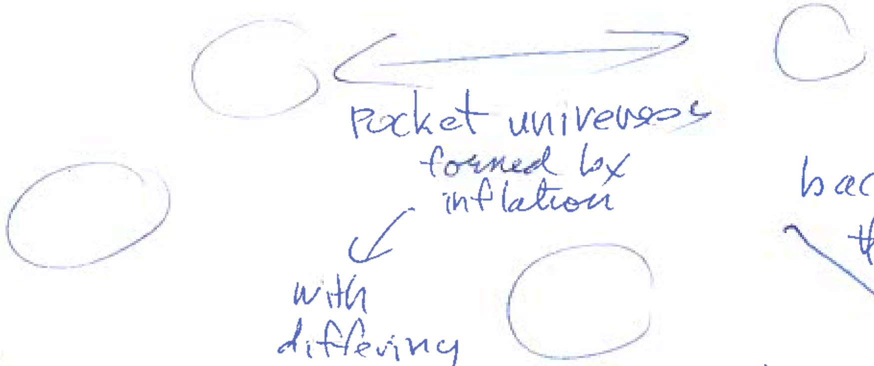
This view is usually considered part of multiverse theory

3096

3100d

All in expansion or contraction regions if all obeys General relativ

— maybe overall expansion or contraction but not pocket univers



Pocket universes formed by inflation

with differing low energy physics and effective Λ varying.

background false vacuum universe → obeys GR to and is always expanding

so pocket universes almost never expand into each other

We'll discuss later when we very qualitatively consider inflation.

Many people think it a reasonable theory — though very hard to prove maybe impossible → But never say die.

But dark energy could just be a spectrum of ~~[illegible]~~ matter baryonic or non-baryonic dark matter

Like my J_5 or J_4

~~In~~ Which case ~~degenerate~~ ~~to my J_A~~ creation

but then it would clump → but would we or large scale structure calculations need that? Not so far.

Bayesian evidence is against

$= -1/3$ ✓ R_{et} vanishes of Helic

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left[\rho - \frac{1}{3}\rho\right]$$

$$\dot{\rho} = -2\frac{\dot{a}}{a}\rho$$

re

$$\rho \propto \frac{1}{a^2}$$

Actually curvature also goes as $\Omega_k \propto \frac{1}{a^2}$ density-like See p. 3072

$$a = a_0\left[\sqrt{\Omega_x}\left(t/t_{H_0}\right)\right]^1$$

$= 1$ for flat universe ? as it must be ; No we could include curvature and matter in this case both go as $\rho \propto \frac{1}{a^2}$

yr $a \propto t$

accelerated universe

see p. 3086 for t_Λ and t_c

e of $w = -1$

$$-3\frac{\dot{a}}{a}\left[\overset{=0}{\rho_M} + \overset{=0}{\rho_r} + \rho_\Lambda + \frac{P_m}{c^2} + \frac{P_r}{c^2} + \frac{P_\Lambda}{c^2}\right] = 0$$

$) + \frac{P_\Lambda}{c^2}$

$= 0, \dot{\rho} = 0$

$\sim \frac{1}{3}\rho_r \to 0$ $\sim -\rho_\Lambda$

but now negligible

DM after radiation era

$\simeq .3$

$\simeq .7$ but note from p. 3071 $\rho_\Lambda \equiv \frac{\Lambda}{8\pi G}$ from a cosmological constant

So is ρ_Λ dark energy with $P_\Lambda = -\rho_\Lambda c^2$ (negative pressure)

$t(t)$

cases

xo d

And we do it in detail in Lect. 5 notes

~~point~~

$k = 0$

verse (1932)

917

1917

FE

$)]^{2/3}$

0

p. 3076.

a is like this

or Cahill 5

~~308~~ 9
3100

or is it Not really dark energy but a cosmological constant.

Recall Einstein field eqn.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G T}{c^4}{}_{\mu\nu}$$

or J

creation of spectrum of matter

here a change in gravity or geometry of universe.

But if you move it to the RHS it becomes dark energy with negative pressure.

or J

~~[illegible]~~

also are effectively a -ve pressure to be li the GR

in context of Friedmann eqn. and $a(t)$ a degeneracy.

But in other contexts maybe there is a distinction (but beyond our course)

No historical impact. Rediscovered much later?

Historical tidbit, none other than Erwin Schrödinger in 1918 ~~[illegible]~~ 8 years before discovering the Schrödinger equation suggested that you move $\Lambda g_{\mu\nu}$ to the RHS

Not then a famous person – discovered over 1929 christmas ski trip ~~only~~ → only vacation he wor… on Schr… sev… abo… vac…

3000i)

and make it a $P = -\rho_c$ term

Schrödinger invents dark energy before 1930s and Zwicky's observational evidence for dark matter

(Raifertaigh 2019 p.13)

~~We'll consider the~~ ΛCDM model with its 3 epochs

Radiation Era 0-51 kyr (Cahill-9)

Matter 51 kyr to 10.1 Gyr

or $t \sim 3.7$ Gyr lookback ago (Cahill-9)

$t_0 = 10.1$ Gyr → Λ era dark energy or whatever

~~What of $\dot\rho \sim 0$~~

~~$p > 0$~~

~~and $\dot a \neq 0$~~ like steady state universe

$0 = -3\frac{\dot a}{a}\rho + \mu$ $\frac{M}{V}$ dimensionless source

$[\dot a \rho] = \frac{M}{TV}$ constants ∴ $H = \frac{\dot a}{a}$ is a constant.

Liddle - 3.5 (p.27) (see 3080a 30[illegible]

Acceleration equation

or 2nd Friedmann equation.

$\frac{k}{a^2}$ $k \sim$

$kc^2 \sim T^{-2} \rightarrow k \sim L^{-2}$

Recall $$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

$$\dot a^2 = \frac{8\pi G}{3}\rho a^2 - kc^2$$ where now $\rho = \sum_i \rho_i$ all density types

$\dot a^2 =$ ~~$\frac{8\pi G(\rho}{3} - kc^2$~~ [illegible]

differentiate

the k term vanishes (and any other $\frac{\rho_0}{a^2}$ energy)

$\Lambda = 8\pi G\rho_\Lambda$ turns it into a ~~density~~ ~~quant~~ constant density quantity or maybe just a constant density

$$2\dot a\ddot a = \frac{8\pi G}{3}\left(\dot\rho a^2 + \rho(2a\dot a)\right)$$

$$\ddot a = \frac{8\pi G}{3}\left(\dot\rho\frac{a^2}{\dot a} + 2\rho a\right)$$

$$\dot\rho = -\frac{3\dot a}{a}\left(\rho + \frac{p}{c^2}\right) +$$ 3096 ~~from p. 3088~~

$$\dot\rho\frac{a}{\dot a} = -3\left[\left(\rho + \frac{p}{c^2}\right)\right.$$ ~~3076~~

From RHS

$$\frac{\ddot a}{a} = \frac{4\pi G}{3}\left[-3\rho - \frac{3p}{c^2} + 2\rho\right]$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\left[\underbrace{\rho + \frac{3p}{c^2}}_{\sum\rho_i + 3\frac{\sum p_i}{c^2}}\right]$$

more like a driver of the motion

This term is usually omitted

μ in $\left(\frac{\dot a}{a}\right)$ Replace by Friedmann eqn

~~ρ~~ $3\rho_\Lambda$ dark energy or ~~[illegible]~~ equation

3'102

EoS $\rho_\Lambda = 0$, $\rho \propto \frac{1}{a^3}$

$\frac{\ddot a}{a} = -\frac{4\pi}{3} G[\rho] \rightarrow$ decelerate.

The acceleration equation is ~~a lot~~ redundant to the Friedmann eqn in solving for a(t) (or t(a)) but it does give a direct way to calculate the acceleration $\ddot a$ which is useful in some contexts — for instance fitting piecewise solutions together smoothly.

P<0

y Λ

ven

deceleration

ut

ita

ρ_Λ

$= -\frac{4\pi}{3}G[\rho_M + \rho_\Lambda - 3\rho_\Lambda]$

$= -\frac{4\pi G}{3} G[\rho_M - 2\rho_\Lambda]$

$\sim \frac{1}{a^3}$

— eventually acceleration

Query ~~How would $\mu\nu$ affect~~ evolution ~~in Friedmann equation.~~ ~~with other~~ when not steady state

~~$\dot\rho = -3\frac{\dot a}{a}(\rho + \frac{P}{c^2})$~~ $+ \mu\nu$ (eq. 3085)

~~say $P = 0$~~

~~$\dot\rho = -3\frac{\dot a}{a}\rho + \mu\nu$~~

~~constant~~

$\dot\rho +$ ~~$3\frac{\dot a}{a}\rho = \mu\nu$ now an inhomogeneous~~ equalu