ION TRAP Quantum Computers

In order to build a quantum computer we need something that can serve as a qubit. It should behave as a quantum mechanical system but be scalable. Several candidates for possible quantum computer applications have been advanced. They include

Ion - trap quantum computing (Cirac-Zoller protocol)
NMR (Nuclear Magnetic Resonance) quantum computers
Adiabatic quantum computing - Dwave
Photonic quantum computers
Cavity QED quantum computers etc.

We shall discuss the Cirac-Zoller scheme which treats a trapped atomic ion as the fundamental qubit.

A working quantum computer should allow individual qubits to be individually addressed, allow two or more qubits to "talk" to each other (in order to built gates), and be scalable. The ion-trap quantum computer has already met these three criteria.

The linear rf-ion trap
You may remember from your physics 181 that the electric potential of a cylinder, held at a constant potential $V_0$ has the form

$$V(x,y) = V_0 + \lambda \log \left( \frac{R}{\rho} \right) \quad \text{for} \; \rho > R$$

$$= V_0 \quad \text{for} \; \rho \leq R$$

where $\rho = \sqrt{x^2 + y^2}$ is the distance from the axis along the center of the cylinder, $R$ is the radius of the cylinder and $\lambda$ is a constant. Let's make a 3D plot of this potential.
\[ V_0 = 2; \]
\[ v[x_, y_, x0_, y0_, R0_] := \text{Which[} \sqrt{((x-x0)^2+(y-y0)^2)} > R0, \]
\[ V_0 + \log[R0/\sqrt{((x-x0)^2+(y-y0)^2)}], \sqrt{((x-x0)^2+(y-y0)^2)} \leq R0, V_0] \]

\[ \text{Plot3D}[v[x, y, 0, 0, 0.5], \{x, -10, 10\}, \{y, -10, 10\}, \text{PlotPoints} \to 100, \text{PlotRange} \to \text{All}] \]
\[ \text{dipole}[x_, y_] := v[x, y, 0, 2, 0.5] - v[x, y, 0, -2, 0.5] \]
\[ \text{Plot3D}[\text{dipole}[x, y], \{x, -10, 10\}, \{y, -10, 10\}, \text{PlotPoints} \to 100, \text{PlotRange} \to \text{All}] \]
\[ \text{quadrupole}[x_, y_] := \]
\[ v[x, y, 0, 2, 0.5] - v[x, y, -2, 0, 0.5] + v[x, y, 0, -2, 0.5] - v[x, y, 2, 0, 0.5] \]
\[ \text{Plot3D}[\text{quadrupole}[x, y], \{x, -4, 4\}, \{y, -4, 4\}, \text{PlotPoints} \to 100, \text{PlotRange} \to \text{All}] \]

A more care full analysis shows that near the origin the potential has the form

\[ V(x,y) = V_0 \left( x^2 - y^2 \right) \Phi_0/2 \text{ where } \Phi_0 \text{ is a constant.} \]
\[ \text{Vquad}[x_, y_] = V_0 \left( x^2 - y^2 \right) \]
\[ 2 \left( x^2 - y^2 \right) \]
\[ \text{Plot3D}[\text{Vquad}[x, y], \{x, -4, 4\}, \{y, -4, 4\}] \]

Now, instead of a constant voltage \( V_0 \) on the electrodes, we can change the polarity in a time-depen- dent way, lets consider

a Cos[\( \omega \) t] dependence so that now the potential at the origin looks like,

\[ V(x,y) = V_0 \cos(\omega t) \left( x^2 - y^2 \right) \Phi_0/2 \]
\[ V_{\text{quad}}[x_-, y_-, t_] = V_0 \cos[\omega t] (x^2 - y^2) \/. \omega \to 2 \pi \]

\[ 2 (x^2 - y^2) \cos[2 \pi t] \]

Manipulate[
  Plot3D[Vquad[x, y, t], {x, -4, 4}, {y, -4, 4}, PlotRange -> {-30, 30}, {t, 0, 2}]

So how does a charged particle behave under the influence of such a "rotating saddle" potential? The force that a particle experiences is the negative of the gradient of the potential times its charge \( q \)

\[ F_x = -q \times V_0 \cos(\omega t) \]

\[ F_y = q \times V_0 \cos(\omega t) \]

and Newton's equation of motion are

\[ m \ddot{x} = -q \times V_0 \cos(\omega t) \]

\[ m \ddot{y} = q \times V_0 \cos(\omega t) \]

\[ \text{eq1} = x''[t] = -q/m \times x[t] \cos[\omega t] \/. \{q \to 1, m \to 1, \omega \to 2 \pi \}
\]

\[ \text{eq2} = y''[t] = q/m \times y[t] \cos[\omega t] \/. \{q \to 1, m \to 1, \omega \to 2 \pi \}
\]

(* initial conditions *)

\[ x_0 = 2.0; \]

\[ y_0 = -1.0; \]

\[ v0x = 1.0; \]

\[ v0y = 1.0; \]

\[ x''[t] = -\cos[2 \pi t] \times x[t] \]

\[ y''[t] = \cos[2 \pi t] \times y[t] \]

\[ \text{tend} = 10; \]

\[ \text{sols} = \text{Flatten}[\text{NDSolve}[[\text{eq1, eq2, x[0] \[Rule] x0, y[0] \[Rule] y0, x'[0] \[Rule] v0x, y'[0] \[Rule] v0y},
\]

\[ \{x[t], y[t], (t, 0, \text{tend})\]]; \]

\[ \text{fig1} = \text{ParametricPlot}[\text{Evaluate}[[x[t], y[t]] \/. \text{sols}], (t, 0, \text{tend}), \text{PlotRange} \to \text{All}] \]

Analytic studies of the above equations show that the dynamics can be described by two types of motion, one is called a micromotion and is evidenced above by the rapidly oscillating wiggles in the overall motion. More important is the overall average motion that is described by a simple 2D harmonic oscillator potential that has the form

\[ V(x,y) = 1/2 \ m \ \Omega^2 (x^2 + y^2) \]

\[ \Omega = \frac{-q \phi_0}{m \omega \sqrt{2}} \]

this potential "binds" the system to the vicinity of the origin. We can analytically solve such a 2D system, the result is
\[ x(t) = A_x \cos(\Omega t + \phi_x) \]
\[ y(t) = A_y \cos(\Omega t + \phi_y) \]

where the quantities \( A_x, A_y, \phi_x, \phi_y \) are related to the initial conditions of the ions motion

\[ x(0) = x_0 = A_x \cos(\phi_x) \]
\[ y(0) = y_0 = A_y \cos(\phi_y) \]

and \( v_x(0) = v_{ox} = -\Omega A_x \sin(\phi_x) \) \( v_y(0) = v_{oy} = -\Omega A_y \sin(\phi_y) \) and so \( \tan(\phi_x) = -v_{ox} / \Omega x_0 \)
\[ \tan(\phi_y) = -v_{oy} / \Omega y_0 \]
\[ A_x = x_0 / \cos(\phi_x) \]
\[ A_y = y_0 / \cos(\phi_y) \]

\[
\text{Omega} = q / m / \omega / \text{Sqrt}[2] /. \{q \to 1, m \to 1, \omega \to 2 \pi\};
\]
\[
\text{phix} = \text{ArcTan}[-v_{0x} / x_0 / \text{Omega}];
\]
\[
\text{phiy} = \text{ArcTan}[-v_{0y} / y_0 / \text{Omega}];
\]
\[
A_x = x_0 / \cos(\text{phix});
\]
\[
A_y = y_0 / \cos(\text{phiy});
\]
\[
\text{xmotion}[t_] = A_x \cos[\text{Omega} t + \text{phix}];
\]
\[
\text{ymotion}[t_] = A_y \cos[\text{Omega} t + \text{phiy}];
\]

\[
\text{ion}[t_0] := \text{Show}[\text{fig1, Graphics}[\{\text{PointSize}[0.05], \text{Point}[[\text{xmotion}[t], \text{ymotion}[t]]],
\text{PlotRange} \to \{[-1.1 A_x, 1.1 A_x], [-1.1 A_y, 1.1 A_y] \}, \text{Axes} \to \text{True}]
\]
\[
\text{fig2} = \text{Manipulate}[[\text{ion}[t], \{t, 0, 2 \pi / \text{Omega}]]
\]
\[
\text{Hyperlink}[^\"\http://www.youtube.com/watch?v=XTJznUkAamIY\""]
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